# Measurement of $\mathbf{e}^{+} \mathbf{e}^{-} \rightarrow \mathrm{K}^{+} \mathrm{K}^{-} \boldsymbol{\pi}^{0}$ cross section and observation of a resonant structure 

## BESIII

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AbSTRACT: Based on $e^{+} e^{-}$collision data collected by the BESIII detector at the BEPCII collider at center-of-mass energies from 2.000 to 3.080 GeV , a partial-wave analysis is performed for the process $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$. The Born cross section of the process $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ and its subprocesses $e^{+} e^{-} \rightarrow \phi \pi^{0}, K^{*+}(892) K^{-}$and $K_{2}^{*+}(1430) K^{-}$ are measured. The results for $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ and $\phi \pi^{0}$ are consistent with the BaBar measurements and with improved precision. By analyzing the cross sections of the subprocesses $e^{+} e^{-} \rightarrow K^{*+}(892) K^{-}$and $K_{2}^{*+}(1430) K^{-}$, a structure with mass $M_{R}=(2190 \pm 19$ $\pm 37) \mathrm{MeV} / c^{2}$ and width $\Gamma_{R}=(191 \pm 28 \pm 60) \mathrm{MeV}$ is observed with a combined statistical significance of $7.1 \sigma$. The measured resonance parameters suggest it can be identified as the $\phi(2170)$, thus the results provide valuable input to understand the internal nature of this state.

Keywords: $e^{+}-e^{-}$Experiments, Particle and Resonance Production, Spectroscopy

ArXiv EPRINT: 2202.06447

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## 1 Introduction

The vector meson state $Y(2175)$, denoted as $\phi(2170)$ by the Particle Data Group (PDG) [1], is one of the most interesting particles in the field of light hadron spectroscopy. The $\phi(2170)$ was first observed by BaBar [2] and subsequently studied at the Belle, BESII and BESIII experiments [3-14]. Possible interpretations of the $\phi(2170)$ state include a conventional $3^{3} S_{1}$ or $2^{3} D_{1} s \bar{s}$ state [15-18], an $s \bar{s} g$ hybrid [16, 19, 20], a tetraquark state [2124], a $\Lambda \bar{\Lambda}\left({ }^{3} S_{1}\right)$ bound state [25-27], or a $\phi K \bar{K}$ resonance state [28, 29]. The branching fractions of $\phi(2170)$ to various final states are critical probes to discriminate the different interpretations of $\phi(2170)$. For instance, the process $\phi(2170) \rightarrow K_{2}^{*+}(1430) K^{-}$has a branching fraction an order less than the branching fraction of $\phi(2170) \rightarrow K_{1}^{+}(1400) K^{-}$ under the hybrid hypothesis $[16,19,20]$, whereas under the conventional state hypothesis the two processes have branching fractions of the same order [15-18].

Experimentally, the $\phi(2170)$ has been studied widely in $e^{+} e^{-}$collisions with final states as $\phi \eta[14,30], \phi \eta^{\prime}[13], \phi f_{0}(980)[2-8], K^{+} K^{-}[9], K^{*+}(892) K^{*-}(892)$ [11], and other $K^{+} K^{-}$states. Among the above final states, none of them is found to be the dominant decay mode, and the products of the $e^{+} e^{-}$partial width and the branching fraction of each final state are all less than 10 eV . The BaBar collaboration has studied the $K_{2}^{*+}(1430) K^{-}$ final state [30] by performing a Dalitz plot analysis of the process $e^{+} e^{-} \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}$ with initial state radiation (ISR) events. The corresponding production cross section has been extracted and no structure has been found around 2.0 GeV [30]. Additionally, the measured ratio of partial decay widths $\Gamma(\phi \eta)[14,30]$ and $\Gamma\left(\phi \eta^{\prime}\right)$ [13] disfavors the hybrid

| $\sqrt{s}(\mathrm{GeV})$ | $\mathcal{L}\left(\mathrm{pb}^{-1}\right)$ | $\sqrt{s}(\mathrm{GeV})$ | $\mathcal{L}\left(\mathrm{pb}^{-1}\right)$ |
| :--- | :---: | :--- | :---: |
| 2.000 | 10.1 | 2.396 | 66.8 |
| 2.050 | 3.4 | 2.644 | 33.6 |
| 2.100 | 12.2 | 2.646 | 34.0 |
| 2.125 | 108.5 | 2.900 | 105.0 |
| 2.150 | 2.8 | 2.950 | 15.9 |
| 2.175 | 10.6 | 2.981 | 16.1 |
| 2.200 | 13.7 | 3.000 | 15.9 |
| 2.232 | 11.8 | 3.020 | 17.3 |
| 2.309 | 22.1 | 3.080 | 126.2 |
| 2.386 | 22.6 |  |  |

Table 1. Integrated luminosities of experimental data.
interpretation [19, 20]. However, the conclusion can be understood by the mechanism of hadronic transition of a strangeonium-like meson along with the $\eta-\eta^{\prime}$ mixing [31]. Therefore, more precise measurements of $\phi(2170)$ 's decay properties are desired to reveal the internal nature of $\phi(2170)$.

In this work, we present a Partial Wave Analysis (PWA) of the process $e^{+} e^{-} \rightarrow$ $K^{+} K^{-} \pi^{0}$ using data collected with the BESIII detector at center-of-mass (c.m.) energies ranging from 2.000 to 3.080 GeV with a total integrated luminosity of $648 \mathrm{pb}^{-1}$, where the detailed values of c.m. energy and integrated luminosities of each data set are presented in table 1. For convenience, we classify all nineteen data sets into groups I and II: group I includes six data sets with c.m. energies $\sqrt{s}=2.000-2.232 \mathrm{GeV}$, and group II for other data sets with $\sqrt{s}=2.309-3.080 \mathrm{GeV}$. Group I and II are fitted using the resonances from the baseline solutions obtained with $\sqrt{s}=2.125 \mathrm{GeV}$ and $\sqrt{s}=2.396 \mathrm{GeV}$ with the two largest statistics, respectively. For the two groups of data sets, the parameters of intermediate states are fixed, and the magnitude and phase of each process are floating.

## 2 BESIII detector and Monte Carlo simulation

The BESIII detector [32] records symmetric $e^{+} e^{-}$collisions provided by the BEPCII storage ring [33], which operates with a peak luminosity of $1 \times 10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$ in the center-ofmass energy range from 2.0 to 4.946 GeV . BESIII has collected large data samples in this energy region [34]. The cylindrical core of the BESIII detector covers $93 \%$ of the full solid angle and consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a $\mathrm{CsI}(\mathrm{Tl})$ electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T ( 0.9 T in 2012) magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identification modules interleaved with steel. The charged-particle momentum resolution at $1 \mathrm{GeV} / c$ is $0.5 \%$, and the $d E / d x$ resolution is $6 \%$ for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of $2.5 \%$
$(5 \%)$ at 1 GeV in the barrel (end cap) region. The time resolution in the TOF barrel region is 68 ps , while that in the end cap region is 110 ps .

A Monte Carlo (MC) simulation based on Geant4 [35], including the geometric description of the BESIII detector and its response, is used to optimize the event selection criteria, estimate backgrounds, and determine the detection efficiency. The signal MC samples are generated using the package ConExc [36], which incorporates a higher-order ISR correction. Background samples of the processes $e^{+} e^{-} \rightarrow e^{+} e^{-}, \mu^{+} \mu^{-}$and $\gamma \gamma$ are generated with the Babayaga [37] generator, while $e^{+} e^{-} \rightarrow$ hadrons and two photon events are generated by the Luarlw [38] and Bestwogam [39] generators, respectively. Signal MC events are generated by using the amplitude model with parameters fixed to the PWA results.

## 3 Event selection and background analysis

The signal process under study is $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ with $\pi^{0} \rightarrow \gamma \gamma$. Thus, candidate events with two oppositely charged kaons and at least two photons are selected. Charged tracks detected in the MDC are required to be within a polar angle $(\theta)$ range of $|\cos \theta|<0.93$, where $\theta$ is defined with respect to the $z$-axis, and their distance of closest approach to the interaction point (IP) must be less than 10 cm along the $z$-axis and less than 1 cm in the transverse plane. Information from TOF and $d E / d x$ measurements is combined to form particle identification (PID) likelihoods for the $\pi, K$, and $p$ hypotheses. Each track is assigned a particle type corresponding to the hypothesis with the highest PID likelihood. Exactly two oppositely charged kaons are required in each event. Photon candidates are identified using showers in the EMC. The deposited energy of each shower is more than 25 MeV in the barrel region $(|\cos \theta|<0.80)$ and more than 50 MeV in the end cap region $(0.86<|\cos \theta|<0.92)$. To exclude showers induced by charged tracks, the angle between the position of each shower in the EMC and the closest extrapolated charged track is required being greater than $10^{\circ}$. To suppress electronic noise and showers unrelated to the event, the difference between the EMC time and the event start time is required to be within $(0,700)$ ns.

To improve the kinematic resolution and suppress background, a four-constraint (4C) kinematic fit imposing energy-momentum conservation with four degrees of freedom is carried out under the hypothesis $e^{+} e^{-} \rightarrow K^{+} K^{-} \gamma \gamma$. If there are more than two photons, the $\gamma \gamma$ combination with minimum $\chi_{4 C}^{2}$ is kept for further analysis. The candidate events are required to satisfy $\chi_{4 C}^{2}<65$. To suppress the contamination from the $e^{+} e^{-} \rightarrow \gamma_{\mathrm{ISR}} \phi$ process, an additional 4C kinematic fit under the hypotheses of $e^{+} e^{-} \rightarrow K^{+} K^{-} \gamma$ is performed. The events are discarded if the corresponding $\chi_{4 C}^{2}$ with any photon inside the event is less than the $\chi_{4 C}^{2}$ of the signal hypothesis. Signal photons are required to have a $M_{\gamma \gamma}$ to be within the $\pi^{0}$ mass region of $[0.120,0.150] \mathrm{GeV} / c^{2}$. The events in the $\pi^{0}$ mass sideband region, defined as $[0.080,0.0115]$ and $[0.160,0.190] \mathrm{GeV} / c^{2}$, are used to estimate the potential backgrounds. After applying the above selection criteria, detailed studies with MC and $\pi^{0}$ sideband indicate that the remaining background contributions are less than $1 \%$ and negligible.

## 4 Amplitude analysis

### 4.1 Partial wave analysis method

Using the GPUPWA framework [40], a PWA is performed on the surviving candidate events to identify the intermediate processes presented in $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$. The amplitude for the $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ decay is constructed with quasi two-body resonances using covariant tensor amplitudes [41]. The intermediate states are parameterized with the relativistic Breit-Wigner (BW) functions. To include the resolution effect for the narrow $\phi$ resonance, a Gaussian function is convolved with the BW function. The resolution effect is negligible for the other resonances, since they have a relatively larger width. The probability is characterized by the measured four-momenta of the particles in the final state [42]. The relative magnitudes and phases of the individual intermediate processes are determined by performing an unbinned maximum likelihood fit using MINUIT [43], where the magnitude and phase of the reference amplitude $e^{+} e^{-} \rightarrow K_{2}^{*+}(1430) K^{-}$are fixed to 1 and 0 , respectively, while those of other amplitudes are floating. Throughout the paper, charge conjugated processes are also included by default.

The PWA fit procedure begins by including all possible intermediate states in the PDG that match $\mathrm{J}^{\mathrm{PC}}$ conservation in the subsequent two-body decay. These intermediate states can decay into $K^{+} K^{-}$or $K^{ \pm} \pi^{0}$ final state. After the fit, the statistical significance of each amplitude is evaluated by incorporating the change in likelihood and degree of freedom fits with and without the corresponding amplitude included in the fit. Amplitudes with statistical significance $<5 \sigma$ are dropped. This procedure is repeated until a baseline solution is obtained with only amplitudes having a statistical significance $>5 \sigma$. To consider the effects of amplitudes with lower significance, an alternative model which contains amplitudes with statistical significance $>3 \sigma$ has been studied and the differences are taken into account in the PWA-mode systematic uncertainty.

The above strategy is implemented individually on the data sets collected at $\sqrt{s}=$ 2.125 and 2.396 GeV , which have the largest luminosities and yields among the nineteen data sets. The baseline solution for data at $\sqrt{s}=2.125 \mathrm{GeV}$ includes the decay processes $e^{+} e^{-} \rightarrow \phi \pi^{0}, \rho(1450) \pi^{0}, \phi(1680) \pi^{0}, \rho(1900) \pi^{0}, \rho_{3}(2250) \pi^{0}, K^{*+}(832) K^{-}, K^{*+}(1410) K^{-}$, $K_{2}^{*+}(1430) K^{-}, K_{3}^{*+}(1780) K^{-}$. In the above, the three-body decays are treated as consecutive quasi two-body decays with a very broad resonance decaying into $K^{+} K^{-}$or $K^{ \pm} \pi^{0}$ and modeled as a $1^{-}$phase space distribution. The statistical significances of all intermediate processes at the two energy points are summarized in table 2. The masses and widths of the $K_{2}^{*+}(1430), \phi(1680)$ and $\rho(1900)$ are determined by scanning the likelihood value in the fit. The measured resonance parameters are consistent with the PDG values within uncertainty, except for the width of $K_{2}^{*}(1430)$ which is within 2 standard deviations. The masses and widths of other intermediate states are fixed to the PDG values. The resonant parameters and fit fractions of the intermediate states are summarized in table 3 and table 4, respectively.

The invariant mass spectra and angular distributions in data and fit results are shown in figures 1 and 2, respectively. The $\chi^{2} / n b i n$ value is displayed on each figure to demonstrate

| Process | Significance $(2.125 \mathrm{GeV})$ | Significance $(2.396 \mathrm{GeV})$ |
| :--- | :---: | :---: |
| $\phi \pi^{0}$ | $18.6 \sigma$ | $2.3 \sigma$ |
| $\rho(1450) \pi^{0}$ | $7.8 \sigma$ | $2.4 \sigma$ |
| $\phi(1680) \pi^{0}$ | $19.5 \sigma$ | $14.9 \sigma$ |
| $\rho(1900) \pi^{0}$ | $7.2 \sigma$ | $7.4 \sigma$ |
| $\rho_{3}(2250)$ | $5.5 \sigma$ | $5.0 \sigma$ |
| $K^{*}(892) K$ | $15.9 \sigma$ | $15.6 \sigma$ |
| $K^{*}(1410) K$ | $5.7 \sigma$ | $5.0 \sigma$ |
| $K_{2}^{*}(1430) K$ | $35.4 \sigma$ | $25.3 \sigma$ |
| $K_{3}^{*}(1780) K$ | $5.8 \sigma$ | $5.5 \sigma$ |

Table 2. Statistical significances of possible intermediate processes at $\sqrt{s}=2.125$ and 2.396 GeV .

| States | Mass $\left(\mathrm{MeV} / c^{2}\right)$ | Width (MeV) | PDG Mass $\left(\mathrm{MeV} / c^{2}\right)$ | PDG Width (MeV) |
| :--- | :---: | :---: | :---: | :---: |
| $K_{2}^{*}(1430)$ | $1428 \pm 2$ | $107 \pm 4$ | $1427.3 \pm 1.5$ | $100.0 \pm 2.1$ |
| $\phi(1680)$ | $1673 \pm 5$ | $172 \pm 8$ | $1680 \pm 20$ | $150 \pm 50$ |
| $\rho(1900)$ | $1880 \pm 10$ | $69 \pm 15$ | $1860-1910$ | $10-160$ |
| $\phi$ | fixed | fixed | $1019.5 \pm 0.02$ | $4.2 \pm 0.01$ |
| $\rho(1450)$ | fixed | fixed | $1465 \pm 25$ | $400 \pm 60$ |
| $\rho_{3}(2250)$ | fixed | fixed | $2248_{-17-5}^{+17+59}[44]$ | $185_{-26-103}^{+31+17}[44]$ |
| $K^{*}(892)$ | fixed | fixed | $891.7 \pm 0.3$ | $50.8 \pm 0.9$ |
| $K^{*}(1410)$ | fixed | fixed | $1414 \pm 15$ | $232 \pm 21$ |
| $K_{3}^{*}(1780)$ | fixed | fixed | $1776 \pm 7$ | $159 \pm 21$ |

Table 3. Masses and widths of the intermediate states at $\sqrt{s}=2.125 \mathrm{GeV}$. Due to the limited data sample size, only the statistical uncertainties are provided. The parameters of $\rho_{3}(2250)$ are cited from ref. [44], where the first uncertainty is statistical and the second one is systematic.

| Process | Fraction (\%) $(2.125 \mathrm{GeV})$ | Fraction (\%) $(2.396 \mathrm{GeV})$ |
| :--- | :---: | :---: |
| $\phi \pi^{0}$ | $1.8 \pm 0.4$ | $0.7 \pm 0.3$ |
| $\rho(1450) \pi^{0}$ | $3.8 \pm 0.7$ | $0.2 \pm 0.2$ |
| $\phi(1680) \pi^{0}$ | $14.6 \pm 2.3$ | $13.6 \pm 2.9$ |
| $\rho(1900) \pi^{0}$ | $2.1 \pm 0.3$ | $3.0 \pm 1.0$ |
| $\rho_{3}(2250)$ | $0.9 \pm 0.5$ | $0.9 \pm 0.6$ |
| $K^{*}(892) K$ | $2.8 \pm 0.3$ | $9.3 \pm 1.2$ |
| $K^{*}(1410) K$ | $1.1 \pm 0.8$ | $3.6 \pm 1.4$ |
| $K_{2}^{*}(1430) K$ | $73.0 \pm 3.7$ | $64.6 \pm 3.2$ |
| $K_{3}^{*}(1780) K$ | $1.3 \pm 0.5$ | $2.1 \pm 1.4$ |

Table 4. Fit fractions of possible intermediate processes at $\sqrt{s}=2.125$ and 2.396 GeV .
the goodness of fit, where nbin is the number of bins of each figure and $\chi^{2}$ is defined as:

$$
\begin{equation*}
\chi^{2}=\sum_{i=1}^{n b i n} \frac{\left(n_{i}-\nu_{i}\right)^{2}}{n_{i}}, \tag{4.1}
\end{equation*}
$$

where $n_{i}$ and $\nu_{i}$ are the number of events for the data and the fit projections in the ith bin of each figure, respectively.

### 4.2 Cross section measurement

The total Born cross section for $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ is obtained at the individual c.m. energy by using:

$$
\begin{equation*}
\sigma^{\mathrm{B}}=\frac{N^{\text {sig }}}{\mathcal{L}_{\text {int }} \cdot \frac{1}{|1-\Pi|^{2}} \cdot(1+\delta)^{\mathrm{r}} \cdot \mathcal{B} r \cdot \epsilon}, \tag{4.2}
\end{equation*}
$$

where $N^{\text {sig }}$ is the corresponding signal yield, which is the number of surviving events due to the negligible background; $\mathcal{L}_{\text {int }}$ is the integrated luminosity; $(1+\delta)^{\mathrm{r}}$ is the ISR correction factor obtained from QED calculations [36, 45] by incorporating the input cross section from this analysis iteratively; $\frac{1}{|1-\Pi|^{2}}$ is the vacuum polarization (VP) factor taken from QED calculations [46]; $\epsilon$ is the detection efficiency obtained from weighting MC simulation according to the PWA results; $\mathcal{B} r$ is the branching ratio of the decay $\pi^{0} \rightarrow$ $\gamma \gamma$ quoted from the PDG [1]. Meanwhile, the Born cross sections for the intermediate processes are obtained with the same approach, individually, while the signal yield $N^{\text {sig }}$ is replaced with the product of the total number of surviving events and the corresponding fraction relative to the total signal yields obtained according to the PWA results, and $\mathcal{B} r$ is replaced with the product of the branching ratio of the decay $\pi^{0} \rightarrow \gamma \gamma$ and that of the intermediate states quoted from the PDG [1]. The number of events of the intermediate processes, $e^{+} e^{-} \rightarrow \phi \pi^{0}, K^{*+}(892) K^{-}$and $K_{2}^{*+}(1430) K^{-}$, are extracted in the PWA fit to calculate the cross section. The measured cross sections as well as the signal yields


Figure 1. At $\sqrt{s}=2.125 \mathrm{GeV}$, (a) invariant mass distribution of $K^{+} K^{-}$; (b) invariant mass distribution of $K^{ \pm} \pi^{0}$; (c) $\cos \theta$ distribution of $K^{+}$in the $K^{+} K^{-}$rest frame; (d) $\cos \theta$ distribution of $K^{+}$in the $K^{+} \pi^{0}$ rest frame; (e) $\cos \theta$ distribution of $K^{+}$in the c.m. frame. $\theta$ is polar angle with respect to the $z$-axis. Dots with error bars are data, and the curves are the fit results.


Figure 2. At $\sqrt{s}=2.396 \mathrm{GeV}$, (a) invariant mass distribution of $K^{+} K^{-}$; (b) invariant mass distribution of $K^{ \pm} \pi^{0}$; (c) $\cos \theta$ distribution of $K^{+}$in the $K^{+} K^{-}$rest frame; (d) $\cos \theta$ distribution of $K^{+}$in the $K^{+} \pi^{0}$ rest frame; (e) $\cos \theta$ distribution of $K^{+}$in the c.m. frame. $\theta$ is polar angle with respect to the $z$-axis. Dots with error bars are data, and the curves are the fit results.

| $\sqrt{s}(\mathrm{GeV})$ | $\mathrm{N}^{\text {sig }}$ | $\epsilon(\%)$ | $(1+\delta)^{\mathrm{r}}$ | $\frac{1}{\|1-\Pi\|^{2}}$ | $\sigma^{\mathrm{B}}(\mathrm{pb})$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 2.000 | $400.0 \pm 20.0$ | 23.4 | 0.979 | 1.037 | $168.4 \pm 8.4 \pm 6.6$ |
| 2.050 | $208.0 \pm 14.4$ | 24.5 | 0.962 | 1.038 | $256.3 \pm 17.8 \pm 9.7$ |
| 2.100 | $712.0 \pm 26.7$ | 25.2 | 0.975 | 1.039 | $232.5 \pm 8.7 \pm 9.1$ |
| 2.125 | $5894.0 \pm 76.8$ | 25.2 | 0.988 | 1.039 | $212.2 \pm 2.8 \pm 8.1$ |
| 2.150 | $152.0 \pm 12.3$ | 25.4 | 1.000 | 1.040 | $204.5 \pm 16.6 \pm 8.0$ |
| 2.175 | $504.0 \pm 22.4$ | 25.6 | 1.014 | 1.040 | $178.7 \pm 8.0 \pm 7.0$ |
| 2.200 | $632.0 \pm 25.1$ | 24.9 | 1.029 | 1.040 | $175.1 \pm 7.0 \pm 6.8$ |
| 2.232 | $520.0 \pm 22.8$ | 24.6 | 1.048 | 1.041 | $165.5 \pm 7.3 \pm 6.3$ |
| 2.309 | $800.0 \pm 28.3$ | 23.7 | 1.091 | 1.041 | $136.2 \pm 4.8 \pm 5.3$ |
| 2.386 | $636.0 \pm 25.2$ | 22.8 | 1.128 | 1.041 | $106.5 \pm 4.2 \pm 4.2$ |
| 2.396 | $1736.0 \pm 41.7$ | 22.9 | 1.133 | 1.041 | $97.4 \pm 2.3 \pm 3.8$ |
| 2.644 | $512.0 \pm 22.6$ | 21.5 | 1.205 | 1.039 | $57.2 \pm 2.5 \pm 2.2$ |
| 2.646 | $560.0 \pm 23.7$ | 21.5 | 1.203 | 1.039 | $62.1 \pm 2.6 \pm 2.4$ |
| 2.900 | $864.0 \pm 29.4$ | 21.4 | 1.218 | 1.033 | $31.0 \pm 1.1 \pm 1.2$ |
| 2.950 | $112.0 \pm 10.6$ | 21.5 | 1.225 | 1.029 | $26.3 \pm 2.5 \pm 1.1$ |
| 2.981 | $120.0 \pm 11.0$ | 21.1 | 1.230 | 1.025 | $28.4 \pm 2.6 \pm 1.1$ |
| 3.000 | $124.0 \pm 11.1$ | 20.6 | 1.236 | 1.021 | $30.5 \pm 2.7 \pm 1.2$ |
| 3.020 | $108.0 \pm 10.4$ | 21.0 | 1.242 | 1.014 | $23.9 \pm 2.3 \pm 1.0$ |
| 3.080 | $824.0 \pm 28.7$ | 19.6 | 1.309 | 0.915 | $28.2 \pm 1.0 \pm 1.1$ |

Table 5. The c.m. energy, detection efficiency, radiative correction factor, vacuum polarization factor, measured cross section for the process $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$, where the first uncertainties are statistical, and the second are systematic.
are summarized in tables 5-8, separately for the process $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ and for each individual intermediate process.

### 4.3 Systematic uncertainties for the intermediate states

Two categories of systematic uncertainties are considered in the measurement of the Born cross sections.

The first category includes those associated with the luminosity, track detection, PID, kinematic fit, ISR correction, and the branching fractions of intermediate states. The uncertainty associated with the integrated luminosity is $1 \%$ at each energy point [47]. The uncertainty of the detection efficiency is $1 \%$ for each charged track [9] and photon [48], individually. The PID efficiency uncertainty is $1.0 \%$ for each charged track [9]. The uncertainty related to the kinematic fit is estimated by correcting the helix parameters of the simulated charged tracks to match the resolution [49]. The uncertainty associated with the ISR and VP effect is obtained with the accuracy of the radiation function, which is about $0.5 \%$ [46], and has a contribution from the cross section lineshape, which is estimated by varying the model parameters of the fit to the cross section. All parameters are randomly

| $\sqrt{s}(\mathrm{GeV})$ | $\mathrm{N}^{\text {sig }}$ | $\epsilon(\%)$ | $(1+\delta)^{\mathrm{r}}$ | $\frac{1}{\|1-\Pi\|^{2}}$ | $\sigma^{\mathrm{B}}(\mathrm{pb})$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 2.000 | $32.6 \pm 10.3$ | 17.6 | 1.604 | 1.037 | $22.6 \pm 7.1 \pm 2.3$ |
| 2.050 | $11.3 \pm 4.7$ | 16.6 | 1.734 | 1.038 | $23.3 \pm 9.7 \pm 2.3$ |
| 2.100 | $20.9 \pm 11.8$ | 17.5 | 1.671 | 1.039 | $11.7 \pm 6.6 \pm 1.2$ |
| 2.125 | $106.4 \pm 22.0$ | 18.1 | 1.621 | 1.039 | $6.6 \pm 1.4 \pm 0.7$ |
| 2.150 | $9.0 \pm 3.9$ | 18.8 | 1.572 | 1.040 | $21.3 \pm 9.1 \pm 2.1$ |
| 2.175 | $16.9 \pm 5.5$ | 19.6 | 1.546 | 1.040 | $10.4 \pm 3.4 \pm 1.0$ |
| 2.200 | $11.0 \pm 5.3$ | 19.3 | 1.592 | 1.040 | $5.2 \pm 2.5 \pm 0.5$ |
| 2.232 | $6.8 \pm 3.2$ | 15.9 | 2.027 | 1.041 | $3.5 \pm 1.6 \pm 0.3$ |
| 2.309 | $2.6 \pm 2.2$ | 9.8 | 3.335 | 1.041 | $0.7 \pm 0.6 \pm 0.1$ |
| 2.386 | $11.4 \pm 4.0$ | 10.0 | 3.104 | 1.041 | $3.2 \pm 1.1 \pm 0.4$ |
| 2.396 | $12.6 \pm 4.7$ | 11.0 | 2.860 | 1.041 | $1.2 \pm 0.4 \pm 0.1$ |

Table 6. The c.m. energy, detection efficiency, radiative correction factor, vacuum polarization factor, measured cross section for the process $e^{+} e^{-} \rightarrow \phi \pi^{0}$, where the first uncertainties are statistical, and the second are systematic.

| $\sqrt{s}(\mathrm{GeV})$ | $\mathrm{N}^{\text {sig }}$ | $\epsilon(\%)$ | $(1+\delta)^{\mathrm{r}}$ | $\frac{1}{\|1-\Pi\|^{2}}$ | $\sigma^{\mathrm{B}}(\mathrm{pb})$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 2.000 | $138.5 \pm 38.8$ | 29.7 | 0.769 | 1.037 | $347.6 \pm 97.5 \pm 35.0$ |
| 2.050 | $57.1 \pm 20.2$ | 30.5 | 0.797 | 1.038 | $406.8 \pm 144.1 \pm 41.0$ |
| 2.100 | $413.2 \pm 98.6$ | 30.5 | 0.846 | 1.039 | $764.1 \pm 182.3 \pm 76.9$ |
| 2.125 | $4304.1 \pm 227.1$ | 30.0 | 0.877 | 1.039 | $873.3 \pm 46.1 \pm 88.4$ |
| 2.150 | $132.3 \pm 25.5$ | 29.4 | 0.902 | 1.040 | $1015.9 \pm 196.0 \pm 102.3$ |
| 2.175 | $417.8 \pm 53.1$ | 29.2 | 0.925 | 1.040 | $845.7 \pm 107.6 \pm 85.1$ |
| 2.200 | $441.4 \pm 48.3$ | 28.6 | 0.944 | 1.040 | $691.7 \pm 75.6 \pm 69.6$ |
| 2.232 | $377.4 \pm 42.6$ | 28.3 | 0.963 | 1.041 | $676.2 \pm 76.3 \pm 68.0$ |
| 2.309 | $548.5 \pm 61.8$ | 27.1 | 0.998 | 1.041 | $531.5 \pm 59.9 \pm 31.4$ |
| 2.386 | $344.3 \pm 36.3$ | 26.3 | 1.023 | 1.041 | $328.7 \pm 34.6 \pm 19.4$ |
| 2.396 | $1117.6 \pm 68.1$ | 26.2 | 1.025 | 1.041 | $360.4 \pm 22.0 \pm 21.3$ |
| 2.644 | $319.1 \pm 26.5$ | 23.6 | 1.102 | 1.039 | $211.5 \pm 17.6 \pm 12.5$ |
| 2.646 | $348.7 \pm 35.1$ | 23.4 | 1.103 | 1.039 | $229.8 \pm 23.1 \pm 13.5$ |
| 2.900 | $493.2 \pm 33.2$ | 20.4 | 1.233 | 1.033 | $108.8 \pm 7.3 \pm 6.4$ |
| 2.950 | $51.1 \pm 10.4$ | 19.6 | 1.268 | 1.029 | $75.6 \pm 15.3 \pm 4.5$ |
| 2.981 | $52.6 \pm 10.2$ | 19.4 | 1.286 | 1.025 | $77.4 \pm 14.9 \pm 4.6$ |
| 3.000 | $68.2 \pm 11.6$ | 19.0 | 1.299 | 1.021 | $102.7 \pm 17.5 \pm 6.0$ |
| 3.020 | $42.8 \pm 9.8$ | 18.9 | 1.309 | 1.014 | $59.5 \pm 13.7 \pm 3.5$ |
| 3.080 | $371.7 \pm 23.7$ | 17.6 | 1.352 | 0.915 | $81.4 \pm 5.2 \pm 4.8$ |

Table 7. The c.m. energy, detection efficiency, radiative correction factor, vacuum polarization factor, measured cross section for the process $e^{+} e^{-} \rightarrow K_{2}^{*+}(1430) K^{-}$, where the first uncertainties are statistical, and the second are systematic.

| $\sqrt{s}(\mathrm{GeV})$ | $\mathrm{N}^{\text {sig }}$ | $\epsilon(\%)$ | $(1+\delta)^{\mathrm{r}}$ | $\frac{1}{\|1-\Pi\|^{2}}$ | $\sigma^{\mathrm{B}}(\mathrm{pb})$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 2.000 | $23.1 \pm 10.3$ | 22.5 | 0.990 | 1.037 | $30.0 \pm 13.4 \pm 3.3$ |
| 2.050 | $22.7 \pm 9.3$ | 23.6 | 0.972 | 1.038 | $86.6 \pm 35.6 \pm 9.5$ |
| 2.100 | $22.3 \pm 17.1$ | 23.8 | 0.985 | 1.039 | $22.9 \pm 17.6 \pm 2.5$ |
| 2.125 | $163.2 \pm 19.8$ | 23.7 | 0.997 | 1.039 | $18.6 \pm 2.3 \pm 2.0$ |
| 2.150 | $5.0 \pm 5.3$ | 23.5 | 1.009 | 1.040 | $21.5 \pm 23.0 \pm 2.4$ |
| 2.175 | $13.1 \pm 7.0$ | 23.6 | 1.024 | 1.040 | $14.9 \pm 7.9 \pm 1.6$ |
| 2.200 | $56.0 \pm 13.0$ | 23.3 | 1.038 | 1.040 | $49.4 \pm 11.4 \pm 5.4$ |
| 2.232 | $36.5 \pm 10.8$ | 23.2 | 1.057 | 1.041 | $36.8 \pm 10.8 \pm 4.0$ |
| 2.309 | $58.7 \pm 30.0$ | 22.5 | 1.099 | 1.041 | $31.4 \pm 16.0 \pm 1.7$ |
| 2.386 | $66.9 \pm 16.0$ | 22.1 | 1.134 | 1.041 | $34.5 \pm 8.3 \pm 1.9$ |
| 2.396 | $159.6 \pm 23.5$ | 22.1 | 1.139 | 1.041 | $27.7 \pm 4.1 \pm 1.5$ |
| 2.644 | $14.6 \pm 11.6$ | 21.2 | 1.211 | 1.039 | $5.0 \pm 3.9 \pm 0.3$ |
| 2.646 | $4.2 \pm 8.4$ | 21.2 | 1.210 | 1.039 | $1.4 \pm 2.8 \pm 0.1$ |
| 2.900 | $47.1 \pm 10.2$ | 21.2 | 1.221 | 1.033 | $5.1 \pm 1.1 \pm 0.3$ |
| 2.950 | $7.7 \pm 6.3$ | 20.9 | 1.227 | 1.029 | $5.5 \pm 4.6 \pm 0.3$ |
| 2.981 | $1.8 \pm 3.2$ | 20.9 | 1.233 | 1.025 | $1.3 \pm 2.3 \pm 0.1$ |
| 3.000 | $0.9 \pm 2.3$ | 20.8 | 1.236 | 1.021 | $0.7 \pm 1.6 \pm 0.0$ |
| 3.020 | $1.1 \pm 1.7$ | 20.7 | 1.240 | 1.014 | $0.8 \pm 1.2 \pm 0.0$ |
| 3.080 | $160.9 \pm 15.5$ | 19.6 | 1.271 | 0.915 | $17.0 \pm 1.6 \pm 1.0$ |

Table 8. The c.m. energy, detection efficiency, radiative correction factor, vacuum polarization factor, measured cross section for the process $e^{+} e^{-} \rightarrow K^{*+}(892) K^{-}$, where the first uncertainties are statistical, and the second are systematic.
varied within their uncertainties and the resulting parametrization of the lineshape is used to recalculate $(1+\delta)^{r} \epsilon$ and the corresponding cross section. This procedure is repeated five hundred times and the standard deviation of the resulting cross section is considered as systematic uncertainty. The uncertainty of the $\pi^{0}$ invariant-mass requirement is evaluated by tuning the MC sample for the $\pi^{0}$ mass resolution according to data at $\sqrt{s}=$ 2.125 GeV . The systematic uncertainties from the branching ratios of intermediate states in the subsequent decays are taken from the PDG [1] and propagated.

The second category of uncertainties are associated with the PWA fit. Fits with alternative scenarios are performed, and the changes of signal yields are taken as systematic uncertainties. Uncertainties associated with the BW parametrization are estimated by replacing the constant-width BW with the mass-dependent width. Uncertainties associated with the resonance parameters, which are taken from the PDG and fixed in the fit, are estimated by performing alternative fits with the added constraints that each resonance parameter follows a Gaussian distribution with a width equal to its uncertainty. One thousand fits are performed, and the resulting relative deviations of the signal yields are taken as systematic uncertainties. Uncertainties associated with the additional resonances are estimated by alternative fits including the components $K^{*}(1680) K$ or the $\rho(1700) \pi^{0}$, which resulted being the most significant, even if with a significance less than $5 \sigma$ obtained from data. Uncertainties due to the barrier factor [50-52] are estimated by varying the radius of the centrifugal barrier from 0.7 to 1.0 fm and considering the difference in $\sigma^{\mathrm{B}}$ as the uncertainty. Uncertainties associated with the MC mode for $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ cross section are estimated by the alternative PWA mode including all the components with a significance more than $3 \sigma$.

In the above procedure, the uncertainties associated with the barrier factor, resonance parametrization and additional resonances are strongly affected by the statistics. Thus, those uncertainties of data with $\sqrt{s}=2.125 \mathrm{GeV}$ are assigned to the group I data, while those of data with $\sqrt{s}=2.396 \mathrm{GeV}$ are assigned to the group II data. For the process $e^{+} e^{-} \rightarrow \phi \pi^{0}$, due to the limited statistics at $\sqrt{s}=2.396 \mathrm{GeV}$, the uncertainties obtained at $\sqrt{s}=2.125 \mathrm{GeV}$ are assigned to all the data sets.

Assuming all the sources of systematic uncertainties as independent, the total uncertainties are the quadratic sums of the individual values, as shown in tables $9-12$, where the sources of the uncertainties tagged with '*' are assumed to be $100 \%$ correlated among c.m. energies.

## 5 Fit to the lineshapes

The measured total Born cross sections for $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ and the Born cross sections for the intermediate process $e^{+} e^{-} \rightarrow \phi \pi^{0}$ are shown in figure 3 , and they are consistent with the previous results from BaBar and SND. The cross sections for the processes $e^{+} e^{-} \rightarrow$ $K^{*+}(892) K^{-}$and $K_{2}^{*+}(1430) K^{-}$are shown in figure 4 , where a clear peak between 2.1 GeV and 2.2 GeV is present.

To further examine the observed structure in the distributions of the measured cross sections of processes $e^{+} e^{-} \rightarrow K_{2}^{*+}(1430) K^{-}$and $K^{*+}(892) K^{-}$, a $\chi^{2}$-fit, incorporating the

| Data set | $\mathcal{L}^{*}$ | Pho. $^{*}$ | Track $^{*}$ | PID* $^{*}$ | $\pi^{0}$ Mass* $^{*}$ | Kin. | ISR | Br $^{*}$ | PWA mode | Sum |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.000 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 1.0 | 0.03 | 0.9 | 3.9 |
| 2.050 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 0.8 | 0.03 | 0.9 | 3.8 |
| 2.100 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 1.1 | 0.03 | 0.9 | 3.9 |
| 2.125 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.08 | 0.8 | 0.03 | 0.9 | 3.8 |
| 2.150 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 1.0 | 0.03 | 0.9 | 3.8 |
| 2.175 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 0.9 | 0.03 | 0.9 | 3.9 |
| 2.200 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 0.9 | 0.03 | 0.9 | 3.9 |
| 2.232 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 0.8 | 0.03 | 0.9 | 3.9 |
| 2.309 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 0.8 | 0.03 | 1.2 | 3.9 |
| 2.386 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.8 | 0.03 | 1.2 | 3.9 |
| 2.396 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.3 | 0.6 | 0.03 | 1.2 | 3.9 |
| 2.644 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.7 | 0.03 | 1.2 | 3.9 |
| 2.646 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.7 | 0.03 | 1.2 | 3.9 |
| 2.900 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.5 | 1.0 | 0.03 | 1.2 | 4.0 |
| 2.950 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.5 | 1.0 | 0.03 | 1.2 | 4.0 |
| 2.981 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 1.0 | 0.03 | 1.2 | 4.0 |
| 3.000 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.5 | 1.1 | 0.03 | 1.2 | 4.0 |
| 3.020 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.5 | 1.0 | 0.03 | 1.2 | 4.0 |
| 3.080 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.5 | 1.0 | 0.03 | 1.2 | 4.0 |

Table 9. Systematic uncertainties (in \%) of $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ at each energy point, where the sources of the uncertainties tagged with "*" are assumed to be $100 \%$ correlated among each energy point.

| Data set | $\mathcal{L}^{*}$ | Pho.* $^{*}$ | Track $^{*}$ | PID $^{*}$ | $\pi^{0}$ Mass* | Kin. | ISR | Res. para | Barrier | Add. Res | Br $^{*}$ | Sum |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.000 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 1.0 | 0.9 | 1.1 | 5.6 | 9.3 | 1.0 | 11.6 |
| 2.050 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 1.0 | 0.9 | 1.1 | 5.6 | 9.3 | 1.0 | 11.6 |
| 2.100 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.9 | 0.9 | 1.1 | 5.6 | 9.3 | 1.0 | 11.6 |
| 2.125 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 1.0 | 0.5 | 1.1 | 5.6 | 9.3 | 1.0 | 11.6 |
| 2.150 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 1.0 | 0.8 | 1.1 | 5.6 | 9.3 | 1.0 | 11.6 |
| 2.175 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 1.0 | 0.9 | 1.1 | 5.6 | 9.3 | 1.0 | 11.6 |
| 2.200 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.9 | 0.9 | 1.1 | 5.6 | 9.3 | 1.0 | 11.6 |
| 2.232 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.9 | 0.8 | 1.1 | 5.6 | 9.3 | 1.0 | 11.6 |
| 2.309 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.8 | 1.1 | 5.6 | 9.3 | 1.0 | 11.6 |
| 2.386 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.8 | 1.1 | 5.6 | 9.3 | 1.0 | 11.6 |
| 2.396 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.9 | 0.6 | 1.1 | 5.6 | 9.3 | 1.0 | 11.6 |

Table 10. Systematic uncertainties (in \%) of $e^{+} e^{-} \rightarrow \phi \pi^{0}$ at each energy point, where the sources of the uncertainties tagged with "*" are assumed to be $100 \%$ correlated among each energy point.

| Data set | $\mathcal{L}^{*}$ | Pho.* $^{*}$ | Track $^{*}$ | PID $^{*}$ | $\pi^{0}$ Mass* | Kin. | ISR | Res. para | Barrier | Add. Res | Br $^{*}$ | Sum |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.000 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.9 | 0.6 | 3.4 | 5.5 | 6.3 | 2.4 | 10.1 |
| 2.050 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 1.0 | 0.6 | 3.4 | 5.5 | 6.3 | 2.4 | 10.1 |
| 2.100 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.7 | 3.4 | 5.5 | 6.3 | 2.4 | 10.1 |
| 2.125 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 1.3 | 0.6 | 3.4 | 5.5 | 6.3 | 2.4 | 10.1 |
| 2.150 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.6 | 3.4 | 5.5 | 6.3 | 2.4 | 10.1 |
| 2.175 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.5 | 3.4 | 5.5 | 6.3 | 2.4 | 10.1 |
| 2.200 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.5 | 3.4 | 5.5 | 6.3 | 2.4 | 10.1 |
| 2.232 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.5 | 3.4 | 5.5 | 6.3 | 2.4 | 10.1 |
| 2.309 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 0.5 | 1.5 | 3.4 | 1.2 | 2.4 | 5.9 |
| 2.386 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.5 | 1.5 | 3.4 | 1.2 | 2.4 | 5.9 |
| 2.396 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 0.5 | 1.5 | 3.4 | 1.2 | 2.4 | 5.9 |
| 2.644 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 0.5 | 1.5 | 3.4 | 1.2 | 2.4 | 5.9 |
| 2.646 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.5 | 1.5 | 3.4 | 1.2 | 2.4 | 5.9 |
| 2.900 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.5 | 1.5 | 3.4 | 1.2 | 2.4 | 5.9 |
| 2.950 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.5 | 1.5 | 3.4 | 1.2 | 2.4 | 5.9 |
| 2.981 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.5 | 1.5 | 3.4 | 1.2 | 2.4 | 5.9 |
| 3.000 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.5 | 1.5 | 3.4 | 1.2 | 2.4 | 5.9 |
| 3.020 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.5 | 1.5 | 3.4 | 1.2 | 2.4 | 5.9 |
| 3.080 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.6 | 1.5 | 3.4 | 1.2 | 2.4 | 5.9 |

Table 11. Systematic uncertainties (in \%) of $e^{+} e^{-} \rightarrow K_{2}^{*+}(1430) K^{-}$at each energy point, where the sources of the uncertainties tagged with "*" are assumed to be $100 \%$ correlated among each energy point.

| Data set | L* $^{*}$ | Pho. $^{*}$ | Track $^{*}$ | PID $^{*}$ | $\pi^{0}$ Mass* | Kin. | ISR | Res. para | Barrier | Add. Res | Br $^{*}$ | Sum |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.000 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.9 | 1.0 | 0.8 | 4.4 | 9.3 | 0.3 | 11.0 |
| 2.050 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.8 | 0.8 | 4.4 | 9.3 | 0.3 | 11.0 |
| 2.100 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.1 | 1.1 | 0.8 | 4.4 | 9.3 | 0.3 | 11.0 |
| 2.125 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.8 | 0.8 | 4.4 | 9.3 | 0.3 | 11.0 |
| 2.150 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 1.0 | 0.8 | 4.4 | 9.3 | 0.3 | 11.0 |
| 2.175 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.9 | 0.8 | 4.4 | 9.3 | 0.3 | 11.0 |
| 2.200 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.8 | 0.9 | 0.8 | 4.4 | 9.3 | 0.3 | 11.0 |
| 2.232 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 0.8 | 0.8 | 4.4 | 9.3 | 0.3 | 11.0 |
| 2.309 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 0.8 | 0.9 | 3.2 | 2.4 | 0.3 | 5.6 |
| 2.386 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.8 | 0.9 | 3.2 | 2.4 | 0.3 | 5.6 |
| 2.396 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.7 | 0.6 | 0.9 | 3.2 | 2.4 | 0.3 | 5.6 |
| 2.644 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.7 | 0.9 | 3.2 | 2.4 | 0.3 | 5.6 |
| 2.646 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 0.7 | 0.9 | 3.2 | 2.4 | 0.3 | 5.6 |
| 2.900 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.5 | 1.0 | 0.9 | 3.2 | 2.4 | 0.3 | 5.6 |
| 2.950 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 1.0 | 0.9 | 3.2 | 2.4 | 0.3 | 5.6 |
| 2.981 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.5 | 1.0 | 0.9 | 3.2 | 2.4 | 0.3 | 5.6 |
| 3.000 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 1.1 | 0.9 | 3.2 | 2.4 | 0.3 | 5.6 |
| 3.020 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.6 | 1.0 | 0.9 | 3.2 | 2.4 | 0.3 | 5.6 |
| 3.080 | 1.0 | 2.0 | 2.0 | 2.0 | 0.3 | 0.5 | 1.0 | 0.9 | 3.2 | 2.4 | 0.3 | 5.6 |

Table 12. Systematic uncertainties (in \%) of $e^{+} e^{-} \rightarrow K^{*+}(892) K^{-}$at each energy point, where the sources of the uncertainties tagged with "*" are assumed to be $100 \%$ correlated among each energy point.


Figure 3. The Born cross sections for (a) the process $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ and (b) the intermediate process $e^{+} e^{-} \rightarrow \phi \pi^{0}$. The red squares are from this analysis; the blue dots and the green triangles are from the BaBar [30] and SND [54] experiments, respectively.


Figure 4. The Born cross section and fit curves for (a) (b) $e^{+} e^{-} \rightarrow K^{*+}$ (892) $K^{-}$, and for (c) (d) $e^{+} e^{-} \rightarrow K_{2}^{*+}(1430) K^{-}$, corresponding to the two solutions in table 13. Dots with error bars are data, where errors include both statistical and systematic uncertainties. The solid (black) curves represent for the total fit result, the short-dashed (red) curves for the intermediate state and the long-dashed (blue) curves for non-resonant component.
correlated and uncorrelated uncertainties among different energy points, is performed. The fit probability density function (PDF) is a coherent sum of a non-resonant component $f_{1}$ and a resonant component $f_{2}$ :

$$
\begin{equation*}
\mathcal{A}=f_{1}+e^{i \varphi} f_{2}, \tag{5.1}
\end{equation*}
$$

where $\varphi$ is the relative phase between the two components. The non-resonant component includes the contributions from phase space (PHSP) and low-mass resonances, and is described as [53].

$$
\begin{equation*}
f_{1}=C_{0} \cdot \sqrt{P S(\sqrt{s})} e^{-p_{0}\left(\sqrt{s}-M_{\mathrm{th}}\right)} \tag{5.2}
\end{equation*}
$$

where $\operatorname{PS}(\sqrt{s})$ is the PHSP distribution, $C_{0}$ and $p_{0}$ are free parameters, $M_{\text {th }}$ is the mass threshold, $M_{\mathrm{th}}=m_{K}+m_{K_{2}^{*}(1430)}$ for $K_{2}^{*+}(1430) K^{-}$and $M_{\mathrm{th}}=m_{K}+m_{K^{*}(892)}$ for $K^{*+}(892) K^{-}$. Here, the relative orbital angular momentum in the two-body decay, $L=2$ for the process $e^{+} e^{-} \rightarrow K_{2}^{*+}(1430) K^{-}$and $L=1$ for the process $e^{+} e^{-} \rightarrow K^{*+}(892) K^{-}$, is considered in the $\operatorname{PS}(\sqrt{s})$ [41] as it follows:

$$
\begin{equation*}
P S(\sqrt{s})=\int\left|A_{K^{*+}(892) K^{-}, K_{2}^{*+}(1430) K^{-}}\right|^{2} d \Phi_{3}, \tag{5.3}
\end{equation*}
$$

where $A$ is partial wave amplitude in the covariant Rarita-Schwinger tensor formalism [41], $\Phi_{3}$ is three-body phase space. The amplitudes for $K_{2}^{*+}(1430) K^{-}$and $K^{*+}(892) K^{-}$are described as:

$$
\begin{align*}
A_{\mu, K_{2}^{*+}(1430) K^{-}} & =-\epsilon_{\mu \nu \lambda \sigma} p_{\psi}^{\sigma} \widetilde{T}_{K_{2}^{*+}(1430) K^{-}}^{(2) \nu \alpha} \cdot f_{K^{+} \pi^{0}}^{K_{2}^{*+}} \cdot \widetilde{t}_{K^{+} \pi^{0} \alpha}^{(2) \lambda}  \tag{5.4}\\
A_{\mu, K^{*+}(892) K^{-}} & =-\epsilon_{\mu \nu \lambda \sigma} p_{\psi}^{\sigma} \widetilde{T}_{K^{*+}(892) K^{-}}^{(1) \nu} \cdot f_{K^{+} \pi^{0}}^{K^{*+}} \cdot \widetilde{t}_{K^{+} \pi^{0}}^{(1) \lambda} \tag{5.5}
\end{align*}
$$

where $\widetilde{T}, \widetilde{t}$ are the covariant tensors, $f$ is a Breit-Wigner propagator [41], $\epsilon_{\mu \nu \lambda \sigma}$ is the Levi-Civita symbol, the other operators are found in reference [41].

The resonant amplitude $f_{2}$ is described with a BW function,

$$
\begin{align*}
f_{2} & =\frac{\sqrt{12 \pi \mathcal{B} r \Gamma_{R}^{e+e^{-}} \Gamma_{R}}}{s-M_{R}^{2}+i M_{R} \Gamma(\sqrt{s})} \cdot \sqrt{\frac{P S(\sqrt{s})}{P S\left(M_{R}\right)}},  \tag{5.6}\\
\Gamma(\sqrt{s}) & =\Gamma_{R} \frac{P S(\sqrt{s})}{P S\left(M_{R}\right)} \tag{5.7}
\end{align*}
$$

where $M_{R}$ is the mass of the resonance, $\Gamma_{R}$ is the constant width, $\Gamma_{R}^{e^{+} e^{-}}$is its partial width to $e^{+} e^{-}$, and $\mathcal{B} r$ is the decay branching fraction to a given final state.

A simultaneous fit, assuming the same resonant structures in the $e^{+} e^{-} \rightarrow K_{2}^{*+}(1430) K^{-}$ and $K^{*+}(892) K^{-}$processes, is performed to the measured cross sections. In the fit, $M_{R}$ and $\Gamma_{R}$ are shared parameters between the two processes and floating, while the production $\mathcal{B} r \Gamma_{R}^{++} e^{-}$and the relative phase angle $\varphi$ are independent between two processes. The fit yields the destructive and constructive solutions with equal fit quality and identical $M_{R}=(2190 \pm 19) \mathrm{MeV} / c^{2}$ and $\Gamma_{R}=(191 \pm 28) \mathrm{MeV}$. The fit curves are shown in figure 4, and the results are summarized in table 13. The overall significance of this resonance is estimated to be $7.1 \sigma$ for the $e^{+} e^{-} \rightarrow K_{2}^{*+}(1430) K^{-}$and $K^{*+}(892) K^{-}$processes, by

| channel |  | $\mathcal{B}_{r} \Gamma_{R}^{e^{+} e^{-}}(\mathrm{eV})$ | $\varphi(\mathrm{rad})$ | Sig. ( $\sigma)$ |
| :--- | :--- | :---: | :---: | :---: |
| $K_{2}^{*+}(1430) K^{-}$ | solution1 | $12.6 \pm 2.4$ | $2.6 \pm 0.2$ | 6.1 |
|  | solution2 | $161.1 \pm 20.6$ | $4.9 \pm 0.1$ |  |
| $K^{*+}(892) K^{-}$ | solution1 | $1.0 \pm 0.3$ | $2.8 \pm 0.3$ | 3.7 |
|  | solution2 | $7.1 \pm 0.9$ | $1.8 \pm 0.1$ |  |

Table 13. A summary of fit results.
comparing the change of $\chi^{2}\left(\Delta \chi^{2}\right)$, with and without the resonant structure in the fit and taking the change of degrees of freedom into account. The significances of the resonant state for the two individual processes are also estimated and summarized in table 13.

The systematic uncertainties on the resonant parameters come from the absolute c.m. energy measurement, the measured cross section, and the fit procedure. The uncertainty of the c.m. energy from BEPCII is small and is ignored in the determination of the parameters of the structure. The statistical and systematic uncertainties of the measured cross section are incorporated in the fit, thus no further uncertainty is necessary. The uncertainties associated with the fit procedure include those from the signal model. To assess the systematic uncertainty associated with the signal model, an alternative BW function with constant width is implemented in the fit, and the resulting differences of $32 \mathrm{MeV} / c^{2}$ and 46 MeV in mass and width, respectively, are considered as the related systematic uncertainties. The uncertainty of the parametrization of the non-resonant component contribution is estimated by changing the term $e^{-p_{0}\left(\sqrt{s}-M_{\text {th }}\right)}$ in eq. (5.2) with $1 / s^{n}$, where $n$ is a free parameter. The differences of the obtained mass and width, which are $17 \mathrm{MeV} / c^{2}$ and 38 MeV , respectively, are assigned as the corresponding systematic uncertainties. The overall systematic uncertainties are the quadratic sum of the individual ones, $37 \mathrm{MeV} / c^{2}$ and 60 MeV for the mass and width, respectively.

## 6 Summary

In summary, a PWA of the process $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ is performed for nineteen data samples with c.m. energies between 2.000 and 3.080 GeV and a total integrated luminosity of $648 \mathrm{pb}^{-1}$ taken by the BESIII detector. The Born cross section of $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$, as well as those for the intermediate processes $e^{+} e^{-} \rightarrow \phi \pi^{0}, K^{*+}(892) K^{-}$and $K_{2}^{*+}(1430) K^{-}$, are measured by performing a PWA on each data sample individually with two baseline solutions according to its c.m. energies. The cross section for $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}$ and $\phi \pi^{0}$ is measured with improved precision and is consistent with those measured by the BaBar experiment. A structure is observed in the cross section of the intermediate processes $e^{+} e^{-} \rightarrow K^{*+}(892) K^{-}$and $K_{2}^{*+}(1430) K^{-}$, and by performing a simultaneous $\chi^{2}$ fit, the two solutions which were obtained confirmed a resonance with mass $M_{R}=(2190 \pm 19$ $\pm 37) \mathrm{MeV} / c^{2}$, width $\Gamma_{R}=(191 \pm 28 \pm 60) \mathrm{MeV}$, and a significance of $7.1 \sigma$, where the uncertainties are statistical and systematic, respectively. The observed resonance is directly produced in $e^{+} e^{-}$collisions, thus a $J^{P C}=1^{--}$is assigned. Comparing to the vector mesons
listed in the PDG [1], the mass of the observed resonance is close to those of $\phi(2170)$, $\rho(2150)$ and $\omega(2290)$, and its width is consistent with that of $\phi(2170)$ within uncertainties, but deviates from those of $\rho(2150)$ and $\omega(2290)$ by more than $3 \sigma$.

Assuming the observed structure is $\phi(2170)$, the relative branching ratio

$$
R=\frac{\mathcal{B} r\left(\phi(2170) \rightarrow K_{2}^{*+}(1430) K^{-}\right)}{\mathcal{B} r\left(\phi(2170) \rightarrow K^{*+}(892) K^{-}\right)}
$$

is calculated to be $12.6 \pm 4.5$ and $22.7 \pm 4.1$ for solution 1 and solution 2 , respectively. It is noticeable that the branching fraction of $\phi(2170) \rightarrow K_{2}^{*+}(1430) K^{-}$is significantly larger than that of $\phi(2170) \rightarrow K^{*+}(892) K^{-}$, which could provide more insight into the nature of $\phi(2170)$. Combined amplitude analysis of $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}, e^{+} e^{-} \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}$ and $e^{+} e^{-} \rightarrow K_{S}^{0} K_{L}^{0} \pi^{0}$ could be performed at BESIII to extract isoscalar and isovector cross sections. In the future, larger data samples taken by BESIII [34] and Belle II [55] can be used to analyze the processes $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{0}, e^{+} e^{-} \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}$ and $e^{+} e^{-} \rightarrow K_{S}^{0} K_{L}^{0} \pi^{0}$ via ISR technique. They would provide more information to understand the structure.

## Acknowledgments

The BESIII collaboration thanks the staff of BEPCII and the IHEP computing center and the supercomputing center of USTC for their strong support. This work is supported in part by National Key R\&D Program of China under Contracts Nos. 2020YFA0406400, 2020YFA0406300; National Natural Science Foundation of China (NSFC) under Contracts Nos. 11335008, 11625523, 11635010, 11735014, 11822506, 11835012, 11935015, 11935016, 11935018, 11961141012, 12022510, 12025502, 12035009, 12035013, 12061131003, 11705192, 11975118, 11950410506, 12061131003; the Natural Science Foundation of Hunan Province under Contract Nos. 2019JJ30019 and 2020RC3054; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; Joint Large-Scale Scientific Facility Funds of the NSFC and CAS under Contracts Nos. U1732263, U1832207, U1832103, U2032111, U2032105; CAS Key Research Program of Frontier Sciences under Contract No. QYZDJ-SSW-SLH040; 100 Talents Program of CAS; INPAC and Shanghai Key Laboratory for Particle Physics and Cosmology; ERC under Contract No. 758462; European Union Horizon 2020 research and innovation programme under Contract No. Marie SklodowskaCurie grant agreement No 894790; German Research Foundation DFG under Contracts Nos. 443159800, Collaborative Research Center CRC 1044, FOR 2359, FOR 2359, GRK 214; Istituto Nazionale di Fisica Nucleare, Italy; Ministry of Development of Turkey under Contract No. DPT2006K-120470; National Science and Technology fund; Olle Engkvist Foundation under Contract No. 200-0605; STFC (U.K.); The Knut and Alice Wallenberg Foundation (Sweden) under Contract No. 2016.0157; The Royal Society, U.K. under Contracts Nos. DH140054, DH160214; The Swedish Research Council; U.S. Department of Energy under Contracts Nos. DE-FG02-05ER41374, DE-SC-0012069.

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