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**FACE RECOGNITION BY GREY-LEVEL CO-OCCURRENCE
MATRICES IN HEXAGONAL DIGITAL IMAGE PROCESSING**

*Nazife ÇEVİK**

ABSTRACT

Face Recognition has been an attractive field of research for decades, because face is one of the most useful and deterministic biometrics. Image processing is called square pixel-based image processing since its existence. However, hexagonal image processing, which is based on the idea of designing and processing pixels as hexagons, has been shown to provide significant benefits in terms of time and memory savings. Almost all of the face recognition methods proposed and implemented so far are based on square pixel based image processing. Based on the limited number of studies on face recognition in hexagonal pixel based image processing, a hexagonal image processing based face recognition method is proposed in this study. The method proposed in this study is inspired by Grey Level Co-occurrence Matrices (GLCM), which is one of the most fundamental of square pixel based face recognition methods. The method is named Hex_Direct_GLCM because it is based on the square pixel-based basic GLCM method. Since hardware-based hexagonal pixel-based image processing is not yet available, hexagonal pixel-based equivalents of square pixel-based digital images are artificially created by software. The hexagonal pixel base equivalents of the steps followed in the GLCM method are performed, and then face recognition accuracy performance analysis is performed on different data sets. As presented in the simulation results, the Hex_Direct_GLCM method provides competitive results with high accuracy in terms of face recognition as well as the success in saving resources such as time and space.



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STRUCTURED ABSTRACT

Individuals pose biological characteristics, which distinguish them from others. Biometrics is concerned with these characteristics. Biometrics has received a great deal of attention in recent years, due to its high discriminatory performance in many areas such as surveillance, identification, and human-computer interaction (Zhong & Zhang, 2013) (Guan et al, 2010) (Jain et al, 2000). These characteristics are generally classified under two headings as physiological and behavioral (Jain & Ross, 2008). Voice, typing rhythm, gait are the examples of the physiological characteristics that contain information about the behavioral characteristics of people (Nisenson et al, 2003). Face is one of the most important characteristics used to distinguish people from others. One of the most important reasons why the face is preferred is that face data can be collected remotely through devices such as cameras without the need for any human intervention. (Dubey, 2017) (Jafri & Arabnia, 2009).

Performing pixel-by-pixel comparison between face images is not the ideal way while analyzing the similarity. This is because, a change in illumination during image acquisition or in a person's facial expression, or noise that can be produced by electronic devices or environmental factors, will degrade facial recognition performance changes in the intensity values of pixels. This will cause a deceptive effect on systems that will perform autonomous decomposition. Therefore, in order to overcome these difficulties related to facial recognition which is one of the prominent human-identification methods, researches have been carried out on this subject and will continue to be done (Lei et al, 2011). As the pixel-based comparison is not appropriate, researches have been made on parameters where human faces can be distinguished by autonomous systems.

Texture, which can be expressed as regular or random patterns that repeat over a region is an important characteristic identifying regions of interest in an image (Dong et al, 2015). Furthermore, texture provides significant detail and information for the classification of images, which yields texture classification to play an important role in fields such as engineering, health and scientific researches (Dong et al, 2015). Hence, texture classification has remarkably taken the attraction and focus of the researchers in the last decades (Reddy, 2014).

GLCM is one of the basic and prominent statistical textual feature extraction method (Ali et al, 2011) that has been widely used in various applications (Champion et al, 2014) (Vujasinovic, 2015) (Wenbo, 2015) (Pratiwi, 2015) for texture analysis. GLCM is the matrix that holds the distribution of co-occurring intensity patterns at a given offset over a given image. Second-order statistical (Haralick) features are extracted to analyze the texture of the image which subsequently can be used for classification tasks (Ou et al, 2014). GLCM which is one of the primary sensitive textual descriptors, handles the -arrangement statistics (Vujasinovic et al, 2015) (Adur et al, 2014). Fourteen (Haralick features) second-order statistical features are calculated on the GLCM, which only four of them are independent (Ou et al, 2014) (Ulaby et al, 1986).

The hexagonal-pixel-based image processing (HIP) is claimed to have significant advantages when compared to the ordinary square-pixel-

based image processing (SIP) for decades. However, since all the mathematical, software and hardware background that have been used since the date of beginning of the image processing science have based on square domain, HIP has not gained the attention, which it deserves.

This study is dedicated to realize one of the prominent issues of image processing and biometrics, that is face recognition, on hexagonal domain. It is desired to be a starting point to translate the ordinary SIP based methods and techniques to the hexagonal domain. Thus, GLCM that is one of the firsts of the holistic approaches, is selected as a candidate to be realized on hexagonal domain. This time, rather than considering 0° , 45° , 90° and 135° orientations of the k -adjacent pixel relations, on the square-pixel domain, 0° , 60° and 120° orientations on the hexagonal-pixel domain are taken account. The simulations result clarifies that, promising accuracy performance can be achieved by Hex_Direct_GLCM whilst saving memory and remedying computational complexity.

Keywords: Face recognition, Gray-Level Co-occurrence Matrices, Hexagonal Image Processing.

ALTİGEN SAYISAL GÖRÜNTÜ İŞLEMEDE GRİ-SEVİYE EŞ-OLUŞUM MATRİSLERİ İLE YÜZ TANIMA

ÖZ

Yüz Tanıma, onlarca yıldır çekici bir araştırma alanı olmuştur, çünkü yüz en kullanışlı ve deterministik biyometrilere biridir. Görüntü işleme, varlığından bu yana kare piksel tabanlı görüntü işleme olarak adlandırılır. Bununla birlikte, pikselleri altıgen olarak tasarlama ve işleme fikrine dayanan altıgen görüntü işlemenin, yapılan araştırmalar sonucunda, zaman ve bellek tasarrufu açısından önemli yararlar sağlayabileceği görülmüştür. Şimdiye kadar ki önerilen ve hayata geçirilen yüz tanıma yöntemlerinin hemen hepsi kare piksel tabanlı görüntü işleme üzerine inşa edilmiştir. Altıgen piksel tabanlı görüntü işlemede yüz tanıma üzerine önerilmiş çalışma sayısının azlığına dayanarak, bu çalışmada, altıgen görüntü işleme tabanlı bir yüz tanıma yöntemi önerilmiştir. Bu çalışmada önerilen yöntemde, kare piksel tabanlı yüz tanıma yöntemlerinin en temellerinden biri olan Gri Seviye Eş-Oluşum Matrislerinden (GLCM) esinlenilmiştir. Yöntem, kare piksel tabanlı temel GLCM metoduna dayandığından, Hex_Direct_GLCM adı verilmiştir. Donanımsal olarak, altıgen piksel tabanlı görüntü işleme henüz mevcut olmadığından, kare piksel tabanlı sayısal görüntülerin, altıgen piksel tabanlı karşılıkları yazılım yoluyla yapay olarak oluşturulmuştur. Kare piksel tabanlı GLCM yönteminde takip edilen adımların altıgen piksel tabandaki karşılıkları gerçekleşmiş ve farklı veri setleri üzerinde yüz tanıma doğruluk performans analizi yapılmıştır. Simülasyon sonuçlarında sunulduğu üzere, Hex_Direct_GLCM yöntemi, hem zaman hem de yer gibi kaynakların tasarrufunda sağladığı başarımın yanı sıra, yüz tanıma açısından da yüksek doğruluk oranıyla rekabetçi sonuçlar vermektedir.

Anahtar Kelimeler: Yüz Tanıma, Gri-Seviye Eş-Oluşum Matrisleri, Altıgen Tabanlı Görüntü İşleme.

1. Introduction

Every human has some unique biological characteristics that distinguish them from the others. The measurements and calculations of physical body features is called the biometrics. Biometrics is the fundamental and ordinary way of discrimination of individuals. In another sense, it is what is consulted in the process of people's authorization in computer science. Biometrics have attracted significant focus as well as found massive application areas such as human-computer application, surveillance, law, etc., due to its high discriminative performance among individuals (Omer & Ertugrul, 2017) (Shih, 2013). These human-specific characteristics are mainly categorized as: physiological and behavioral. Biological characteristics such as gait, typing rhythm and voice are the representatives of behavioral characteristics, while face, iris, retina are the physiological ones (Nisenson et al, 2003).

Biometrics recognition is the identification of individuals by means of extracting the abovementioned characteristics and examining them by using machine learning algorithms in computer science. Face is one of the prominent and handy biometrics utilized in recognition, since face data can be gathered by using remote devices, such as cameras, without any human intervention and disturbance (Dubey, 2017) (Jafri & Arabnia, 2009). As in all other types of images, face images are also influenced by many factors such as noise, lighting and exposure changes. Because of the aforementioned factors, sudden changes may occur on some of the pixels. That causes the occasion that even if two separate images belong to the same person, due to the value discrepancies in the individual pixels, the pictures appear as if they belonged to two different people. Consequently, one-to-one pixel comparison is not preferred in biometrical image based recognition.

In such cases where a one-to-one pixel comparison is not possible or unfavorable, there is a need for something that will be least affected by the external factors mentioned earlier; the name of it is the texture. Textures, as abovementioned defined as repeating patterns, are extracted from the image, following that, they are analyzed and compared to discriminate the individuals. In order to qualify a texture as qualified, its computational complexity must be low, its ability to distinguish, and robustness against external performance-degrading factors must be high (Liu et al, 2017).

Due to the increasing importance of biometrics and expansion of its application areas, texture based biometrics recognition has gained significant interest of the researchers. The main focus of the studies about this topic is to meet the three abovementioned requirements, low complexity, high recognition performance, robustness to external factors, as much as possible. These studies are categorized mainly as: holistic and localized (Lei et al, 2011). Holistic descriptors where *GLCM* (Haralick et al, 1973), Basic Component Analysis (PCA) (Turk and Pentland, 1991), Linear Discrimination Analysis (LDA) (Belhumeur et al, 1997), and Independent Component Analysis (ICA) (Comon, 1994) are the most basic, popular, and inspiring ones, address the image holistically and extract the holistic features that represent the general characteristics of the image. The dazzling ability and reputation of PCA is that it enables the transition from the high-dimensional high-correlation-features set space to a lower-dimensional uncorrelated-features set space, while retaining much of the information while doing so. PCA achieves the task of dimension reduction by projecting the data onto a linear surface. Like PCA, LDA deals with finding features on a lower dimensional space that represents the characteristic of the class best which eventually maximizes the inter-class variation and minimizes the intra-class variation. PCA, LDA and ICA have inspired many researchers, and thus, many new ideas based on these basic methods have been proposed. Local descriptors (LBP (Ahonen et al, 2004), LGBP (Zhang et al, 2005), CS-LBP (Heikkilä et al, 2009), GV-LBP (Lei et al, 2011), LDP (Jabid et al, 2010), LJBWP (Dan et al, 2014), PLBP (Qian et al, 2011), LDGP (Chakraborty et al, 2017), LPQ (Yang & Bhanu, 2011), LDNP (Rivera et al, 2013) (Rivera & Chae, 2015), HoG (Dahmane & Meunier, 2011), LTP (Tan & Triggs, 2010), Gabor (Yin & Kim, 2008) (Melendez et al, 2008)) consider local pixel relationships rather than dealing holistic characteristic of the images. Eventually, local descriptors or holistic ones, both are not sufficient in recognition individually. Often, after the extraction of local

patterns, histograms or other parameters are used to investigate how these models are dispersed across the image.

HIP is put forward especially last years and is claimed to enable significant savings in terms of time, complexity and memory. Although there has been some effort on academic studies, cause all mathematical background such as matrices and linear algebra are based on square or rectangular architecture, as well as the hardware components does also, improvements in HIP have not gained a satisfying accelerate. Since there is no standardized and common algebraic background, as well as software package or libraries on HIP, almost all topics such as recognition, edge detection, noise filtering, etc., have been handled on square pixel based domain. Hence, this study is dedicated to realize one of the prominent issues of image processing and biometrics, that is face recognition, on hexagonal domain. It is desired to be a starting point to translate the ordinary SIP based methods and techniques to the hexagonal domain. Thus, *GLCM* that is one of the firsts of the holistic approaches, is selected as a candidate to be realized on hexagonal domain. This time, rather than considering 0° , 45° , 90° and 135° orientations of the k -adjacent pixel relations, on the square-pixel domain, 0° , 60° and 120° orientations on the hexagonal-pixel domain are taken account. The simulations result clarifies that, promising accuracy performance can be achieved by *Hex_Direct_GLCM* whilst saving memory and remedying computational complexity.

The rest of the article is organized as follows. Section II gives a preliminary knowledge about the ordinary *GLCM*, while Section III describes *Hex_Direct_GLCM*. The performance comparison of *Hex_Direct_GLCM* against the ordinary *GLCM* is mentioned in Section IV. Lastly, Section V concludes the article.

2. Methodology

Face recognition methods are mainly classified as holistic and local as mentioned in the previous section. Almost all face recognition studies proposed thus far, focused on SIP. However, the counterparts of these approaches have not been inclined properly. To be a starting point for this matter, in this study, the hexagonal counterpart of one of the fundamental and earliest holistic methods, that is *GLCM* is elaborated. To be a starting point for this matter, in this study, the hexagonal counterpart of *GLCM* that is one of the fundamental and earliest holistic methods, is elaborated. Firstly, a summarized explanation of *GLCM* is given and following that, some preliminary knowledge about hexagonal image processing is given.

2.1. GLCM

Haralick et. al. proposed *GLCM* and some features that are extracted from these matrices for image classification. *GLCMs* hold the neighboring pixels (P_i, P_j) intensity relationships (R_{P_i, P_j}), as well as their occurrence statistics. While calculating the occurrence statistics the whole image is considered. Hence, it is classified as a holistic method. The operational logic of *GLCM* is as follows: let $f_{m \times n}$ be an image with m rows n columns, that consists of pixels that's intensity values take values in the interval $[0, L-1]$, where $0 < L \leq 256$. Each element of *GLCM* refers to the number of times that the pixel pair (P_i, P_j) occurred in f with orientation Q . The orientation represented with Q is eventually depicts a displacement vector $d=(d_x, d_y \mid d_x=d_y=d_g)$ where d_g is the number of gaps between the pixels of interest. For the situation of adjacency, $d_g=0$. Orientation can also be represented with two parameters as the distance d_g that the pixel pair (P_i, P_j) apart from each other with angle α . α can take one of the values 0° , 45° , 90° and 135° . The values that d take, vary depending on α as: for $\alpha=0^\circ$ $0 \leq d_g \leq m-2$, $\alpha=45^\circ$ $0 \leq d_g \leq \min(m,n)$, $\alpha=90^\circ$ $0 \leq d_g \leq n-2$ and $\alpha=135^\circ$ $0 \leq d_g \leq \min(m,n)$ theoretically. Since there are four diverse angles as 0° , 45° , 90° and 135° , four diverse *GLCMs* are produced for each image. *GLCMs* are square matrices having equal sizes. Their sizes depend on the discrete intensity values in the image, as $GLCM_{L-1 \times L-1}$. Figure 1 illustrates the calculation of four *GLCMs* ($GLCM_{0^\circ}$, $GLCM_{45^\circ}$, $GLCM_{90^\circ}$, $GLCM_{135^\circ}$) for a sample image

$f_{8 \times 8}$ of which pixel intensity values vary in the interval $[0-7]$.

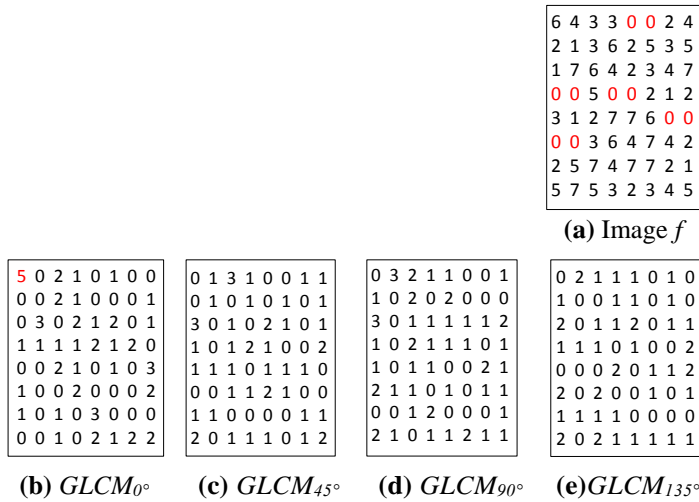


Figure 1. Demonstration of the GLCMs for a sample matrix.

Following the calculation of $GLCM_\alpha$ s, Haralick features are extracted from these matrices. Fourteen that are most popular among these are listed in Table 1.

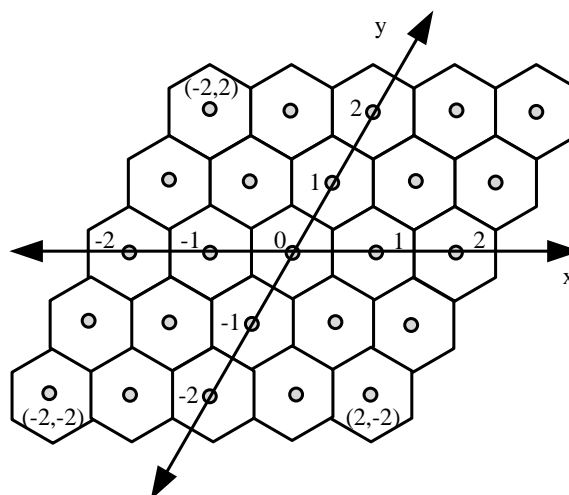
Table 1. Haralick Features.

Angular Second Moment	$f_1 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p(i,j)^2$
Contrast	$f_2 = \sum_{n=0}^{N_g-1} n^2 \left\{ \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p(i,j) \right\}_{ i-j =n}$
Correlation	$f_3 = \frac{\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} (i-\mu_x)(j-\mu_y)p(i,j)}{\sigma_x \sigma_y}$
Sum of Squares	$f_4 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} (i-\mu)^2 p(i,j)$
Inverse Difference Moment	$f_5 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \frac{1}{1+(i-j)^2} p(i,j)$
Sum Average	$f_6 = \sum_{i=2}^{2N_g} i p_{x+y}(i)$
Sum Variance	$f_7 = \sum_{i=2}^{2N_g} (i - f_6)^2 p_{x+y}(i)$
Sum Entropy	$f_8 = - \sum_{i=2}^{2N_g} p_{x+y}(i) \log\{p_{x+y}(i)\}$
Entropy	$f_9 = - \sum_i \sum_j p(i,j) \log\{p(i,j)\}$
Difference Variance	$f_{10} = \text{variance of } p_{x-y}$
Difference Entropy	$f_{11} = - \sum_{i=2}^{N_g-1} p_{x-y}(i) \log\{p_{x-y}(i)\}$
Information Measures of Correlation	$f_{12} = \frac{HXY - HXY1}{\max\{HX, HY\}}$
	$f_{13} = (1 - \exp[-2.0(HXY2 - HXY)])^{1/2}$ $HXY = - \sum_i \sum_j p(i,j) \log p(i,j)$
Maximal Correlation Coefficient	$Q(i,j) = \sum_k \frac{p(i,k)p(j,k)}{p_x(i)p_y(k)}$ $f_{14} = (\text{Second Largest Eigenvalue of } Q)^{1/2}$

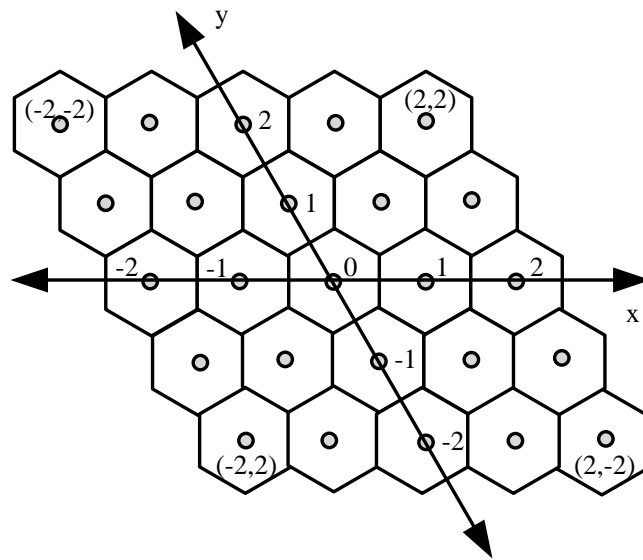
2.2. HIP

Although light signals that contain visual data, are continuous, computers can only process digital data. Therefore, continuous light data should be sampled and digitized. Since square or rectangular sensor arrays are used during light detection, the computer side finishing operations are designed accordingly. Therefore, the smallest data unit of the digitized data on the computer side is designed in the form of pixels. However, sampling of light data on a hexagonal mesh and then maintaining subsequent operations in the hexagonal area can change many things and give promising results. Hexagonal geometry has been explored for many years. There was the assumption that the best way to divide an area into sub-regions with equal regions could be done with hexagons until they were proved by Hales (Hales, 2000) (Hales, 2001). In addition to the natural hexagonal arrangement of the photoreceptors in the fovea, the most beautiful natural example in which the hexagonal geometry is encountered is the honeycombs made by other bees.

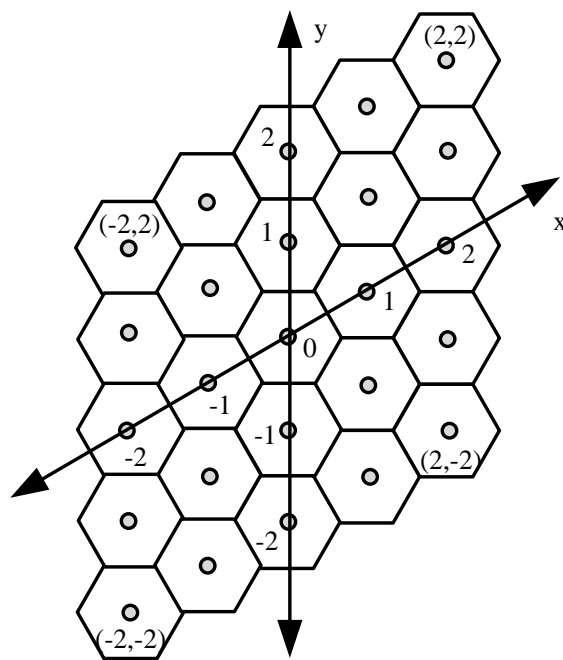
Hexagonal geometry can make significant improvements in image processing. The work done so far has focused on the difficulty of mapping from SIP to HIP, and suggested elegant suggestions on how to represent square pixels in the hexagonal plane. Since the dots on a hexagonal grid are not aligned in two orthogonal directions, it is not always possible to represent the addresses of the dots in a hexagonal grid with Cartesian coordinates. The most prominent and frequently exploited in many studies (Watson & Ahumada, 1989) (Stevenson & Arce, 1985) (Bell et al, 1989) (Staunton, 1999) addressing schemes on hexagonal domain are depicted in Figure 2.



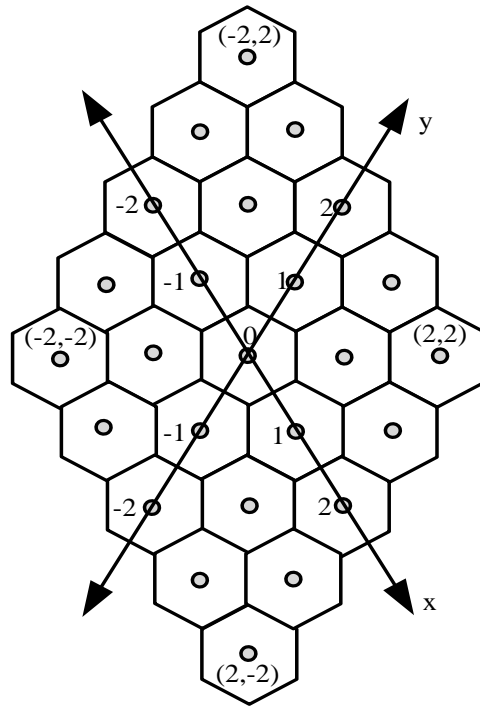
(a)



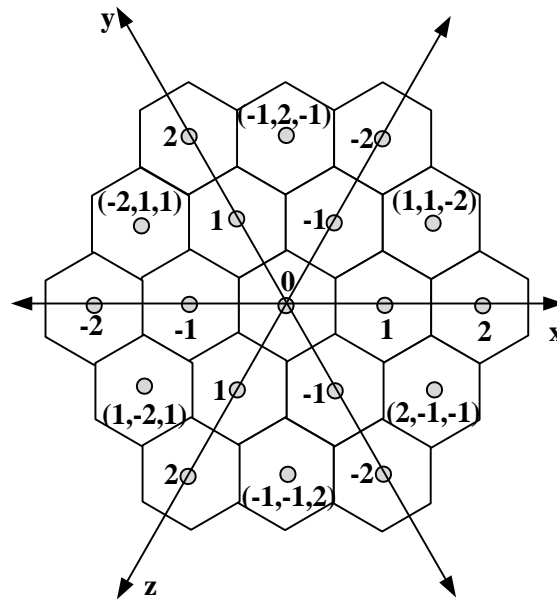
(b)



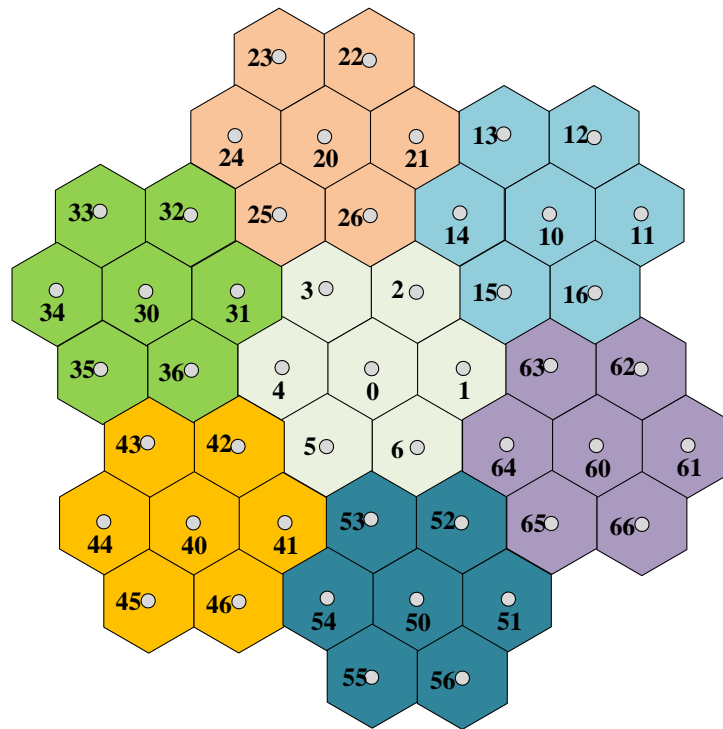
(c)



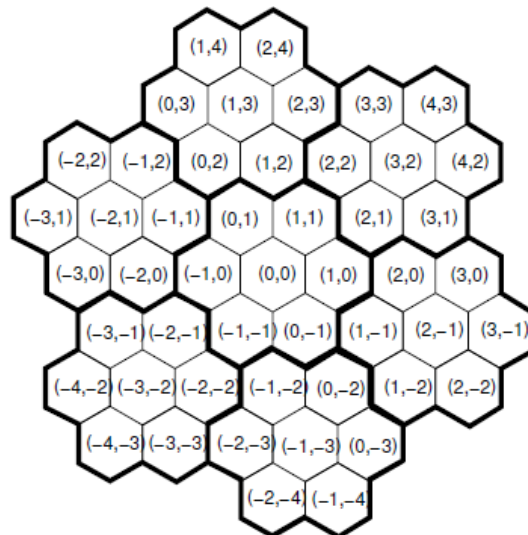
(d)



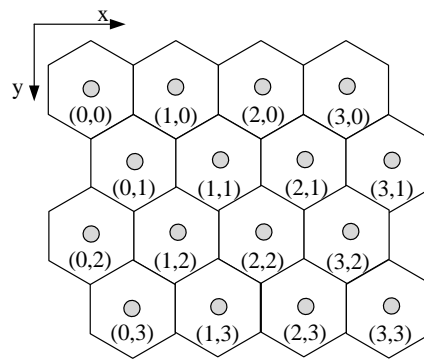
(e)



(f)



(g)



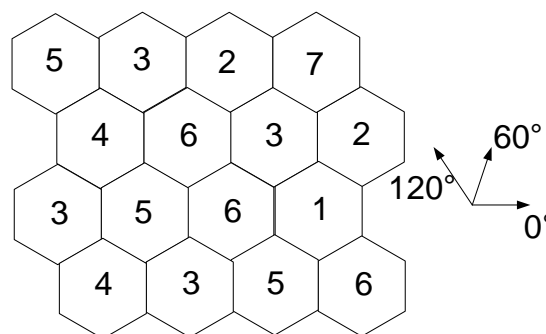
(h)

Figure 2. Addressing methods on hexagonal grid (a-e) Skewed-axes addressing (f-g) Hierarchical addressing (h) Ordinary flat addressing schemes.

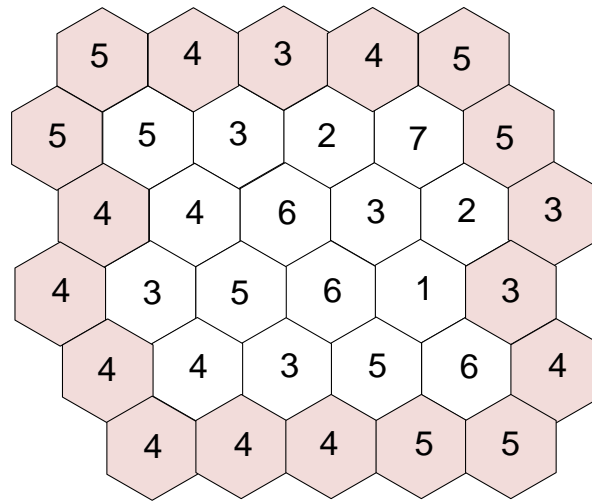
2.3. Hex_Direct_GLCM

Face is one of the prominent biometric type due to its no need of human intervention to gather data. Furthermore, face data based recognition has high discrimination accuracy. *GLCM* based discrimination has been one of the fundamental and inspiring approach and applied to many areas of pattern recognition. However, as all the other methods proposed thus far, *GLCM*-based pattern recognition methods have based on SIP. A counterpart of *GLCM*-based face recognition method has not been proposed thus far. In this article, *Hex_Direct_GLCM* the counterpart of a previously proposed *GLCM*-based descriptor (*Direct_GLCM*) (Eleyan & Demirel, 2011) has been proposed.

As expressed in the previous section, four *GLCMs* ($GLCM_{0^\circ}$, $GLCM_{45^\circ}$, $GLCM_{90^\circ}$, $GLCM_{135^\circ}$) are extracted from an image. However, on hexagonal domain there are three main directions, that are 0° , 60° , 120° . Hence, on hexagonal domain, three *GLCMs* are produced rather than four. Figure 3 depicts the representation of a sample piece of an image and the *GLCM_Hexs* produced from that image. Most of the operations in digital image processing require padding step in advance. That is, all pixels in the image should pose at least 1-hop adjacent neighboring pixels. Hence, for the pixels residing at the sides of the image, two methods are utilized. First method pads the image with replication of the side-pixels. The other method calculates the intensity values of the padding pixels by taking the average of the present adjacent pixels. The padded form of the original image according to the abovementioned padding methods are also given in Figure 3.



(a)



(b)

0	0	1	0	0	0	0
0	0	1	0	0	0	1
0	2	0	0	2	0	0
0	0	1	0	0	1	0
0	0	1	0	0	2	0
1	0	1	1	0	0	0
0	0	0	0	1	0	0

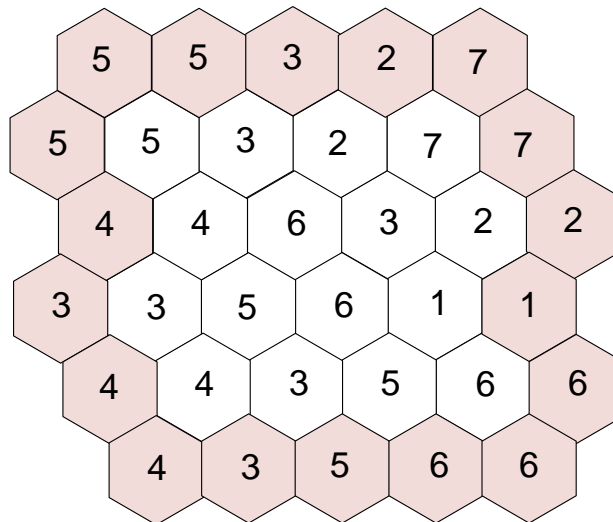
(c)

0	1	0	0	0	0	0
0	0	0	1	1	0	0
0	0	1	1	0	1	1
0	0	1	0	1	0	0
1	0	0	1	0	1	0
0	1	2	0	0	0	0
1	0	0	0	0	0	0

(d)

0	0	1	0	0	0	0
0	0	1	0	0	0	1
0	1	0	2	1	0	0
0	0	1	0	1	0	0
0	0	0	1	1	1	0
1	0	1	0	0	1	0
0	0	0	1	0	0	0

(e)



(f)

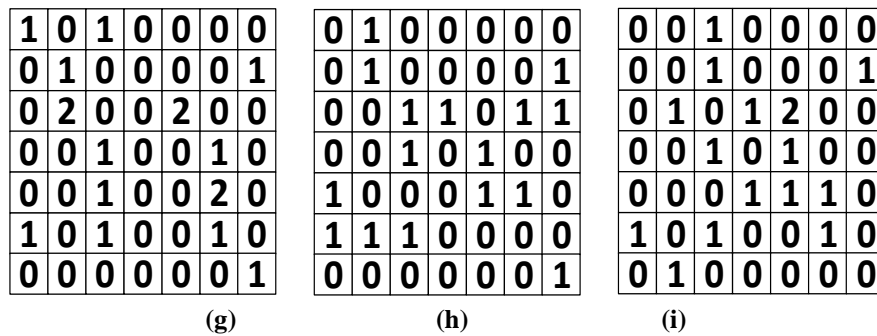


Figure 3. (a) A sample hexagonal representation of a piece of an image f_{hex} (b) The padded image ($f_{hex(rep)}$) by replication (c-e) $GLCM_{Hex0^\circ}$, $GLCM_{Hex60^\circ}$, $GLCM_{Hex120^\circ}$ of $f_{hex(rep)}$. (f)The padded image ($f_{hex(avg)}$) by averaging (g-i) $GLCM_{Hex0^\circ}$, $GLCM_{Hex60^\circ}$, $GLCM_{Hex120^\circ}$ of $f_{hex(avg)}$.

Direct_GLCM applies directly the *GLCMs* extracted from the image as feature vectors rather than extracting Haralick features from them. Hence, since *Hex_Direct_GLCM* is the counterpart of *Direct_GLCM* on the hexagonal domain, *GLCM_Hexs* are extracted directly as feature vectors.

3. Experimental Results

The performance of the proposed method regarding recognition accuracy is done on four datasets, namely: 1) Fac-e94 Dataset (Libor, 2000) - contains 153 distinct individuals each has 20 frontal views of each individual with different facial expressions, 2) ORL dataset (Samaria & Harter, 1994) - contains ten different images of 40 individuals, 3) Japanese Female Facial Expression (JAFFE) dataset (Lyons et al, 1998) - contains 213 images of 7 facial expressions posed by 10 Japanese models 4) CAS-PEAL-R1 dataset (Gao et al, 2008) - is a subset of CAS-PEAL and contains 30,900 images of 1040 subjects with varying Pose, Expression, Accessory, and Lighting. Figure 4 illustrates sample images that are retrieved from the mentioned datasets.

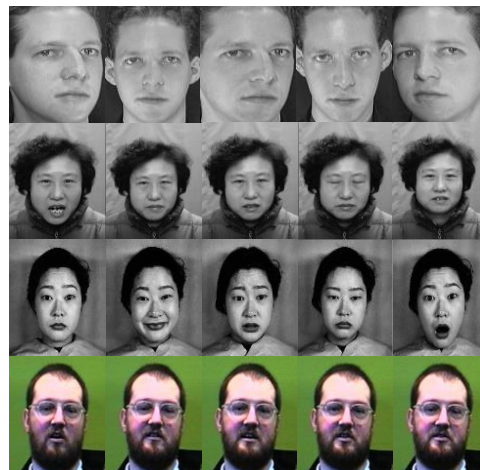


Figure 4. Sample face images extracted from ORL, CAS-PEAL-R1, JAFFE and Face94 datasets respectively.

A plenty of simulations are conducted to compare the recognition accuracy performances of *Hex_Direct_GLCM* and *Direct_GLCM*. Firstly, the recognition accuracy performances are measured when each individual *GLCM* ($GLCM_{0^\circ}$, $GLCM_{45^\circ}$, $GLCM_{90^\circ}$, $GLCM_{135^\circ}$, $GLCM_{Hex0^\circ}$,

$GLCM_{Hex60^\circ}$, $GLCM_{Hex120^\circ}$) is applied individually. At the second step, the *GLCMs* of both

square and hexagonal domains are concatenated separately and a single feature vector is produced for both square and hexagonal domains, namely $GLCM_{ALL}$ and $GLCM_{Hex_{ALL}}$. Table 2 gives the recognition accuracy performances of all abovementioned $GLCMs$ on the defined datasets.

Table 2. Recognition accuracy performances

Method	Dataset			
	Face94	CAS-PEAL-R1	JAFFE	ORL
<i>Hex_Direct_GLCM_Fuse</i>	0.99	0.84	0.78	0.84
<i>Hex_Direct_GLCM_0</i>	0.99	0.84	0.74	0.88
<i>Hex_Direct_GLCM_60</i>	0.99	0.82	0.78	0.85
<i>Hex_Direct_GLCM_120</i>	0.99	0.82	0.76	0.85
<i>Direct_GLCM_Fuse</i>	0.99	0.85	0.78	0.86
<i>Direct_GLCM_0</i>	0.99	0.83	0.76	0.88
<i>Direct_Sq_GLCM_45</i>	0.99	0.81	0.78	0.83
<i>Direct_Sq_GLCM_90</i>	0.99	0.84	0.80	0.85
<i>Direct_Sq_GLCM_135</i>	0.99	0.82	0.76	0.80

As clarified in Table 2, *Hex_Direct_GLCM* the same performance as *Direct_GLCM*. Furthermore, there is no need to fuse the discrete $GLCMs$ to accomplish a higher performance, because each individual $GLCM$ of both the square and hexagonal domains achieve the similar results as the fused one.

4. Conclusion

A plenty of face recognition methods have been proposed thus far. However, almost all of these studies have been designed for the square domain. If the hexagonal based image processing is intended to improved and made prevalent, hexagonal-domain-applicable recognition methods should also become up with. Hence, to be an inspiring and encouraging study, this study introduces the hexagonal counterpart of one of the basic recognition methods that is $GLCMs$. The hexagonal-based $GLCM$ is not less than the traditional one, but has been found to perform better than it at some points. As a future work, it is intended to realize and introduce more of the hexagonal-counterparts of the ordinary square-based recognition methods.

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