Determination of Strong-Phase Parameters in $D \to K^0_{S,L}\pi^+\pi^-$

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We report the most precise measurements to date of the strong-phase parameters between D^0 and $\bar{D}^0$ decays to $K^0_{S,L}\pi^+\pi^-$ using a sample of 2.93 fb^{-1} of e^+e^- annihilation data collected at a center-of-mass energy of 3.773 GeV with the BESIII detector at the BEPCII collider. Our results provide the key inputs for a binned model-independent determination of the Cabibbo-Kobayashi-Maskawa angle γ/ϕ_3 with B decays. Using our results, the decay model sensitivity to the γ/ϕ_3 measurement is expected to be between 0.7° and 1.2°, approximately a factor of three smaller than that achievable with previous measurements, based on the studies of the simulated data. The improved precision of this work ensures that measurements of γ/ϕ_3 will not be limited by knowledge of strong phases for the next decade. Furthermore, our results provide critical input for other flavor-physics investigations, including charm mixing, other measurements of CP violation, and the measurement of strong-phase parameters for other D-decay modes.

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The mechanism of CP violation in particle physics is of primary importance because of its impact on cosmological baryogenesis and matter-antimatter asymmetry in the universe. In the standard model (SM), CP violation is studied by measuring the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1], using the convenient representation given by the unitarity triangle (UT) formed in the complex plane. The angle γ (also denoted ϕ_3) of the UT is of particular interest since it is the only one that can be extracted from tree-level processes, for which the contribution of non-SM effects is expected to be very small. Therefore, measurement of γ provides a benchmark for the SM with minimal theoretical uncertainty [2,3]. A precision measurement of γ is an essential ingredient in comprehensive testing of the SM description of CP violation and probing for evidence of new physics. Direct measurements of γ have not yet achieved the required precision, with a world-average value of $\gamma = (73.5^{+4.2}_{-5.1})^\circ$ [4], to be compared to the indirect determination of $\gamma = (65.8^{+1.0}_{-1.7})^\circ$ [5]. These different determinations deviate by 1.5σ. It has been predicted that new physics at the tree level could introduce a deviation in γ up to 4° [6], which is close to the current experimental precision. Achieving subdegree precision in the determination of γ is clearly a top priority for current and future flavor-physics experiments.

Generally, three methods had been suggested to measure γ so far: GLW [7,8], ADS [9,10], and Dalitz (GGSZ) [11] analyses. One of the most sensitive decay channels for measuring γ is $B^- \to DK^-$ with $D \to K^0_S\pi^+\pi^-$ [11], where D represents a superposition of D^0 and $\bar{D}^0$ mesons.

(Throughout this Letter, charge conjugation is assumed unless otherwise explicitly noted.) The model-independent approach [12] requires a binned Dalitz plot analysis of the amplitude-weighted average cosine and sine of the relative strong-phase ($\Delta\delta_D$) between D^0 and $\bar{D}^0 \to K_S^0\pi^+\pi^-$ to determine γ. These strong-phase parameters were first studied by the CLEO collaboration using 0.82 fb^{-1} of data [13,14]. The limited precision of CLEO's results contributes a systematic uncertainty of approximately 4° to the γ measurement [15], currently the dominant systematic limitation in this determination. In the coming decades, the statistical uncertainties of measuring γ will be greatly reduced by LHCb and Belle II, potentially to 1° or less. The model-independent approach provides the most precise stand-alone γ measurement [15], and therefore improved measurements of the D strong-phase parameters are essential in maximizing the precision of γ from these future data sets.

In this Letter, we use the model-independent approach of Ref. [12] for the determination of the strong-phase parameters between D^0 and $\bar{D}^0 \to K_{S,L}^0\pi^+\pi^-$. More details are presented in a companion paper [16]. Our data sample was collected from e^+e^- annihilations at $\sqrt{s} = 3.773$ GeV, just above the energy threshold for production of $D\bar{D}$ events. At this energy we take advantage of unique quantum correlations afforded by production through the $\psi(3770)$ resonance. The total integrated luminosity of our sample is 2.93 fb^{-1} [17], 3.6 times that of the CLEO measurement. The expected improvement in precision of the strong-phase parameters will significantly reduce the uncertainties of determinations of γ [15,18–21] that utilize $D \to K_{S,L}^0\pi^+\pi^-$. Additionally, improved knowledge of these strong-phase parameters will have significant impact in other applications, including measurements of the CKM angle β (also denoted ϕ_1) through time-dependent analyses of $B^0 \to Dh^0$ [22] (where h is a light meson) and $B^0 \to D\pi^+\pi^-$ [23], as well as measurements of charm mixing and CP violation [24–27].

For this study we analyze the $D \to K_S^0\pi^+\pi^-$ Dalitz plot phase space of m_-^2 vs m_+^2, where m_-^2 and m_+^2 are the squared invariant masses of the $K_S^0\pi^-$ and $K_S^0\pi^+$, respectively. The phase space is partitioned into eight pairs of irregularly shaped bins following the three schemes defined in Ref. [14], which are divided according to regions of similar strong-phase difference $\Delta\delta_D$ or maximum sensitivity to γ in the presence of negligible (significant) background; here these schemes are referred to as "equal $\Delta\delta_D$" and "(modified) optimal," respectively. The bin index i ranges from −8 to 8 (excluding 0), with the bins symmetric under the exchange $m_-^2 \leftrightarrow m_+^2$ ($i \leftrightarrow -i$). The strong-phase parameters are denoted c_i and s_i, where c_i is the amplitude-weighted average of $\cos\Delta\delta_D$ in the ith region of the Dalitz plot ($\mathcal{D}_i$) and is given by

$$c_i = \frac{\int_{\mathcal{D}_i} |\mathcal{A}||\bar{\mathcal{A}}| \cos\Delta\delta_D d\mathcal{D}}{\sqrt{\int_{\mathcal{D}_i} |\mathcal{A}|^2 d\mathcal{D} \int_{\mathcal{D}_i} |\bar{\mathcal{A}}|^2 d\mathcal{D}}}, \tag{1}$$

where $\mathcal{A}$ and $\bar{\mathcal{A}}$ are the amplitudes for $D^0 \to K_S^0\pi^+\pi^-$ and $\bar{D}^0 \to K_S^0\pi^+\pi^-$, respectively. The term s_i is defined analogously, with $\cos\Delta\delta_D$ replaced by $\sin\Delta\delta_D$. Because the effects of charm mixing and CP violation in the D decay are negligible, we take $c_i = c_{-i}$ and $s_i = -s_{-i}$. The measurement involves studying the density of the correlated $D \to K_S^0\pi^+\pi^-$ vs $D \to K_{S,L}^0\pi^+\pi^-$ Dalitz plots, as well as decays of a D meson tagged in a CP eigenstate decaying to $K_{S,L}^0\pi^+\pi^-$. The expected yields can be expressed in terms of the parameters K_i, c_i, and s_i for $D^0 \to K_S^0\pi^+\pi^-$, and K_i', c_i', and s_i' for $D \to K_L^0\pi^+\pi^-$, where $K_i^{(\prime)}$ is determined from the distribution of the flavor-tagged $D^0 \to K_{S,L}^0\pi^+\pi^-$ decays across the bins of the Dalitz plot as $K_i^{(\prime)} = h_D \int_{\mathcal{D}_i} |\mathcal{A}|^2 d\mathcal{D}$ and h_D is a normalization factor. Therefore, the strong-phase parameters c_i, s_i, c_i', and s_i' can be determined by minimizing the likelihood function constructed from the observed and expected yields of these decays.

Details about the BESIII detector design and performance are provided in Ref. [28]. To measure strong-phase parameters, we select "single-tag" (ST) and "double-tag" (DT) samples as listed in Table I. STs are D mesons reconstructed from their daughter particles in one of 17 decay modes, of which four are flavor specific, five are CP even, seven are CP odd, and one ($K_S^0\pi^+\pi^-$) is CP mixed. Note that we count $D \to \pi^+\pi^-\pi^0$ as a CP-even eigenstate while explicitly correcting for its small CP-odd component [29]. DTs are events with an ST and a second D meson reconstructed as either $K_S^0\pi^+\pi^-$ or $K_L^0\pi^+\pi^-$. The K_L^0 mesons are not directly reconstructed and their presence is inferred by partial reconstruction technique where one particle is identified by the missing energy and mass in the event. DTs are only formed in combinations where there is a maximum of one unreconstructed particle.

The selection and yield determination procedures of ST and DT candidates are described in the companion paper [16] and are summarized below. The ST yields, $N_{\rm ST}$, are listed in the second column of Table I. The yields of DT candidates consisting of $K_S^0\pi^+\pi^-$ vs fully reconstructed final states are determined with a two-dimensional unbinned maximum-likelihood fit to the $M_{\rm BC}^{\rm sig}$ (signal) vs $M_{\rm BC}^{\rm tag}$ (tag) distribution. The DT candidates with an undetectable neutrino or K_L^0 are reconstructed by combining a $K_S^0\pi^+\pi^-$ candidate with the remaining charged or neutral particles, that are assigned to the other D decay. The variable $U_{\rm miss} = E_{\rm miss} - |\vec{p}_{\rm miss}|$ (for $K^+e^-\bar{\nu}_e$) or missing-mass squared ($M_{\rm miss}^2$) are calculated from the missing energy and momentum in the event. To reduce background

TABLE I. Summary of ST yields (N_{ST}) and DT yields for $K^0_{S,L}\pi^+\pi^-$ vs various tags. The uncertainties are statistical only. The tag modes of $\pi^+\pi^-\pi^0$, $K^0_S\eta'$, $K^0_L\pi^0\pi^0$ and the partially reconstructed $K^0_S\pi^+\pi^-$ events are used for the first time.

Mode	N_{ST}	$N^{\mathrm{DT}}_{K^0_S\pi^+\pi^-}$	$N^{\mathrm{DT}}_{K^0_L\pi^+\pi^-}$
Flavor tags			
$K^+\pi^-$	549373 ± 756	4740 ± 71	9511 ± 115
$K^+\pi^-\pi^0$	1076436 ± 1406	5695 ± 78	11906 ± 132
$K^+\pi^-\pi^-\pi^+$	712034 ± 1705	8899 ± 95	19225 ± 176
$K^+e^-\bar{\nu}_e$	458989 ± 5724	4123 ± 75	
CP-even tags			
K^+K^-	57050 ± 231	443 ± 22	1289 ± 41
$\pi^+\pi^-$	20498 ± 263	184 ± 14	531 ± 28
$K^0_S\pi^0\pi^0$	22865 ± 438	198 ± 16	612 ± 35
$\pi^+\pi^-\pi^0$	107293 ± 716	790 ± 31	2571 ± 74
$K^0_L\pi^0$	103787 ± 7337	913 ± 41	
CP-odd tags			
$K^0_S\pi^0$	66116 ± 324	643 ± 26	861 ± 46
$K^0_S\eta_{\gamma\gamma}$	9260 ± 119	89 ± 10	105 ± 15
$K^0_S\eta_{\pi^+\pi^-\pi^0}$	2878 ± 81	23 ± 5	40 ± 9
$K^0_S\omega$	24978 ± 448	245 ± 17	321 ± 25
$K^0_S\eta'_{\pi^+\pi^-\eta}$	3208 ± 88	24 ± 6	38 ± 8
$K^0_S\eta'_{\gamma\pi^+\pi^-}$	9301 ± 139	81 ± 10	120 ± 14
$K^0_L\pi^0\pi^0$	50531 ± 6128	620 ± 32	
Mixed CP tags			
$K^0_S\pi^+\pi^-$	188912 ± 756	899 ± 31	3438 ± 72
$K^0_S\pi^+\pi^-_{\mathrm{miss}}$		224 ± 17	
$K^0_S(\pi^0\pi^0_{\mathrm{miss}})\pi^+\pi^-$		710 ± 34	

contributions, events with excess neutral energy or charged tracks are rejected.

The $K^0_S\pi^+\pi^-$ vs $K^0_S\pi^+\pi^-$ DTs are crucial for determining the s_i values, and thus in order to increase the yield for these events, we include two types of partially reconstructed events, which more than doubles the yield. The first ($K^0_S\pi^\pm\pi^\mp_{\mathrm{miss}}$) allows for one pion originating from the D meson to be unreconstructed in the detector. For these events, which have only three charged tracks recoiling against the $D \to K^0_S\pi^+\pi^-$ ST, the missing pion is inferred from the M^2_{miss} of the event. The second [$K^0_S(\pi^0\pi^0_{\mathrm{miss}})\pi^+\pi^-$] is the case where one K^0_S meson decays to $\pi^0\pi^0$, with only one π^0 detected while the other π^0 is undetected. We select events with only two additional oppositely charged tracks recoiling against the $D \to K^0_S\pi^+\pi^-$ ST and identify these as the π^+ and π^- from the other D meson. The resulting distributions of M^2_{miss} show clear signals with minimal background, and signal yields are obtained with unbinned maximum-likelihood fits, as is shown in Fig. 1.

The DT yields of $K^0_S\pi^+\pi^-$ and $K^0_L\pi^+\pi^-$ tagged by different channels are shown in the third and fourth columns of Table I, respectively. Overall, the DT yields of $D \to K^0_{S(L)}\pi^+\pi^-$ involving a CP eigenstate are a factor

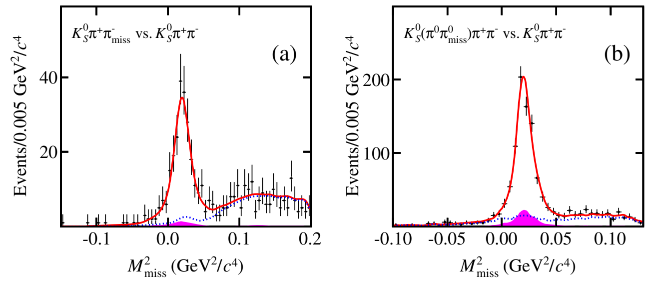

FIG. 1. Fits to M^2_{miss} distributions in data. Points with error bars are data, dotted (blue) curves are the fitted combinatorial backgrounds. The shaded areas (pink) show Monte Carlo (MC) estimates of the peaking backgrounds mainly from (a) $D \to \pi^+\pi^-\pi^+\pi^-$ and (b) $D \to \pi^+\pi^-\pi^0\pi^0$, and the red solid curves are the total fits.

of 5.3 (9.2) larger than those in Ref. [14], and the DT yields of $K^0_S\pi^+\pi^-$ tagged with $D \to K^0_{S(L)}\pi^+\pi^-$ decays are a factor of 3.9 (3.0) larger than those in Ref. [14]. These increases come not only from the larger data set available at BESIII but also from the additional tag modes and the application of partial-reconstruction techniques. Figure 2 shows the Dalitz plots of CP-even and CP-odd tagged $D \to K^0_S\pi^+\pi^-$ events selected in the data. The effect of quantum correlations arising from production through $\psi(3770) \to D^0\bar{D}^0$ is demonstrated by the differences between these plots. Most noticeably, the CP-odd component $K^0_S\rho(770)^0$ is visible in CP-even tagged $K^0_S\pi^+\pi^-$ samples but absent from CP-odd samples.

The DT yield for the ith bin of the Dalitz plot of each tagged $D \to K^0_{S(L)}\pi^+\pi^-$ sample, N_i^{obs}, can be determined by fitting the DT events observed in this bin. Here the yield includes the signal and any peaking background component. The expected DT yields in the ith bin of Dalitz plot of each tagged $D \to K^0_{S(L)}\pi^+\pi^-$ sample, N_i^{exp}, are sums of the expected signal yields and the expected peaking backgrounds. It should be noted that detector resolution effects can cause individual events to migrate between Dalitz plot bins after reconstruction. Such migration effects vary among bins due to the irregular bin shapes, coupled with the rapid variations of the Dalitz plot density. Furthermore, migrations differ between $D \to K^0_S\pi^+\pi^-$ and $D \to K^0_L\pi^+\pi^-$ decays due to different resolutions in the Dalitz plots (0.0068 GeV$^2/c^4$ for $D \to K^0_S\pi^+\pi^-$ and 0.0105 GeV$^2/c^4$ for $D \to K^0_L\pi^+\pi^-$). The resultant bin migrations range within (3–12)% and (3–18)% for the $K^0_S\pi^+\pi^-$ and $K^0_L\pi^+\pi^-$ signals, respectively. Therefore, in the determination of the DT yields, simulated efficiency matrices are introduced to account for bin migration and reconstruction efficiencies [16]. Studies indicate that neglecting bin migration introduces biases in the determination of $c_i(s_i)$ that average a factor of 0.7 (0.3) times the statistical uncertainty of this analysis, so it is important to

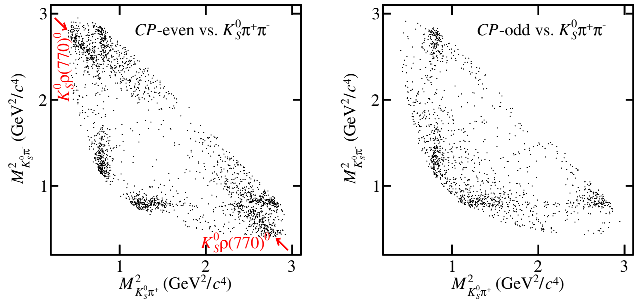

FIG. 2. Dalitz plots of $K_S^0\pi^+\pi^-$ events in data. The effect of the quantum correlation is clearly visible. The approximate locations of events from $K_S^0\rho(770)^0$ are indicated by arrows for clarity.

correct for this effect. The values of K_i and K_i' that are used to evaluate $N_i^{\rm exp}$ are determined from the flavor-tagged DT yields, where corrections from doubly Cabibbo-suppressed decays, efficiency and migration effects have been applied, which are explained in detail in Ref. [16].

The values of $c_i^{(\prime)}$ and $s_i^{(\prime)}$ are obtained by minimizing the negative log-likelihood function constructed as

$$-2\log\mathcal{L} = -2\sum_i\sum_j \ln P(N_{ij}^{\rm obs},\langle N_{ij}^{\rm exp}\rangle)_{K_S^0\pi^+\pi^-,K^0_{S(L)}\pi^+\pi^-} - 2\sum_i \ln P(N_i^{\rm obs},\langle N_i^{\rm exp}\rangle)_{CP,K^0_{S(L)}\pi^+\pi^-} + \chi^2,$$

where $P(N^{\rm obs},\langle N^{\rm exp}\rangle)$ is the Poisson probability to observe $N^{\rm obs}$ events given the expected number $\langle N^{\rm exp}\rangle$. Here the sums are over the bins of the $D^0\to K^0_{S(L)}\pi^+\pi^-$ Dalitz plots. The χ^2 term is used to constrain the difference $c_i'-c_i$ ($s_i'-s_i$) to the predicted quantity Δc_i (Δs_i). The values of Δc_i and Δs_i are estimated based on the decay amplitudes of $D^0\to K_S^0\pi^+\pi^-$ [30] and $D^0\to K_L^0\pi^+\pi^-$, where the latter is constructed by adjusting the $D^0\to K_S^0\pi^+\pi^-$ model taking the K_S^0 and K_L^0 mesons to have opposite CP, as is discussed in Refs. [13,14]. The details of assigning Δc_i (Δs_i) and their uncertainties $\delta\Delta c_i$ ($\delta\Delta s_i$) are presented in Table VI of Ref. [16].

The measured strong-phase parameters $c_i^{(\prime)}$ and $s_i^{(\prime)}$ are presented in Fig. 3 and Table II. The estimation of systematic uncertainties is described in detail in Ref. [16]. In addition to our results, Fig. 3 includes the predictions of Ref. [30] and the results from Ref. [14], which show reasonable agreement.

In summary, measurements of the strong-phase parameters between D^0 and $\bar{D}^0\to K^0_{S,L}\pi^+\pi^-$ in bins of phase space have been performed using 2.93 fb^{-1} of data collected at $\sqrt{s}=3.773$ GeV with the BESIII detector. Compared to the previous CLEO measurement [14], two main improvements have been incorporated. First, additional tag decay modes are used. In particular the inclusion of the $\pi^+\pi^-\pi^0$ tag improves the sensitivity to c_i and the addition of the $K_S^0(\pi^0\pi^0_{\rm miss})\pi^+\pi^-$ improves the sensitivity to s_i. Second, corrections for bin migration have been included, as their neglect would lead to uncertainties comparable to the statistical uncertainty. The results presented in this Letter are on average a factor of 2.5 (1.9) more precise for c_i (s_i) and a factor of 2.8 (2.2) more precise for c_i' (s_i') than has been achieved previously. The strong-phase parameters provide an important input for a wide range of CP violation measurements in the beauty and charm sectors, and also for measurements of strong-phase parameters in other D decays where $D\to K_S^0\pi^+\pi^-$ is used as a tag [31,31–34].

To assess the impact of our c_i and s_i results on a measurement of γ, we use a large simulated data set of $B^-\to DK^-$, $D\to K_S^0\pi^+\pi^-$ events. Based on the MC simulation, the uncertainty in γ associated with our uncertainties for c_i and s_i is found to be 0.7°, 1.2°, and 0.8° for the equal $\Delta\delta_D$, optimal and modified optimal binning schemes, respectively. For comparison, the corresponding results from CLEO are 2.0°, 3.9°, and 2.1° [14]. Therefore,

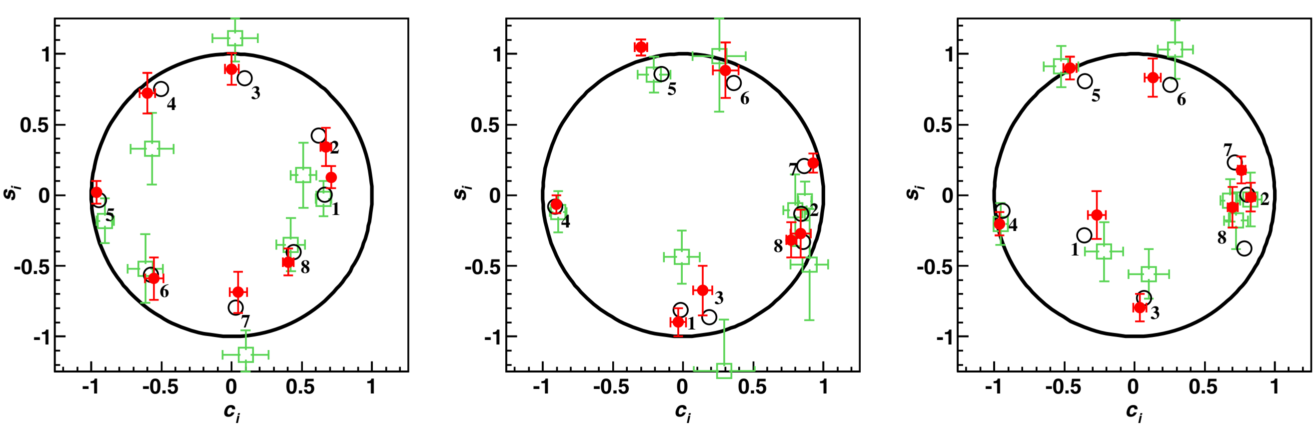

FIG. 3. The c_i and s_i measured in this work (red dots with error bars), the predictions of Ref. [30] (black open circles) and the results of Ref. [14] (green open squares with error bars). The left, middle and right plots are from the equal $\Delta\delta_D$, optimal and modified optimal binnings, respectively. The circle indicates the boundary of the physical region $c_i^2+s_i^2=1$.

TABLE II. The measured strong-phase parameters $c_i^{(\prime)}$ and $s_i^{(\prime)}$, where the first uncertainties are statistical, including that related to the Δc_i and Δs_i constraints, and the second are systematic.

	Equal $\Delta\delta_D$ binning		Optimal binning		Modified optimal binning	
	c_i	s_i	c_i	s_i	c_i	s_i
1	0.708(0.020)(0.009)	0.128(0.076)(0.017)	−0.034(0.052)(0.017)	−0.899(0.094)(0.030)	−0.270(0.061)(0.019)	−0.140(0.168)(0.027)
2	0.671(0.035)(0.016)	0.341(0.134)(0.015)	0.839(0.062)(0.037)	−0.272(0.166)(0.031)	0.829(0.027)(0.018)	−0.014(0.100)(0.018)
3	0.001(0.047)(0.019)	0.893(0.112)(0.019)	0.140(0.064)(0.028)	−0.674(0.172)(0.037)	0.038(0.044)(0.021)	−0.796(0.095)(0.020)
4	−0.602(0.053)(0.016)	0.723(0.143)(0.015)	−0.904(0.021)(0.009)	−0.065(0.062)(0.006)	−0.963(0.020)(0.009)	−0.202(0.080)(0.014)
5	−0.965(0.019)(0.013)	0.020(0.081)(0.009)	−0.300(0.042)(0.013)	1.047(0.055)(0.014)	−0.460(0.044)(0.011)	0.899(0.078)(0.013)
6	−0.554(0.062)(0.024)	−0.589(0.147)(0.030)	0.303(0.088)(0.027)	0.884(0.191)(0.042)	0.130(0.055)(0.017)	0.832(0.131)(0.029)
7	0.046(0.057)(0.023)	−0.686(0.143)(0.028)	0.927(0.016)(0.008)	0.228(0.066)(0.015)	0.762(0.025)(0.012)	0.178(0.094)(0.016)
8	0.403(0.036)(0.017)	−0.474(0.091)(0.027)	0.771(0.032)(0.015)	−0.316(0.123)(0.020)	0.699(0.035)(0.012)	−0.085(0.141)(0.018)
	c_i'	s_i'	c_i'	s_i'	c_i'	s_i'
1	0.801(0.020)(0.013)	0.137(0.078)(0.016)	0.240(0.054)(0.015)	−0.854(0.106)(0.032)	−0.198(0.067)(0.025)	−0.209(0.181)(0.027)
2	0.848(0.036)(0.016)	0.279(0.137)(0.016)	0.927(0.054)(0.036)	−0.298(0.162)(0.029)	0.945(0.026)(0.018)	−0.019(0.100)(0.017)
3	0.174(0.047)(0.016)	0.840(0.118)(0.020)	0.742(0.060)(0.030)	−0.350(0.180)(0.039)	0.477(0.040)(0.019)	−0.709(0.119)(0.028)
4	−0.504(0.055)(0.019)	0.784(0.147)(0.014)	−0.930(0.023)(0.019)	−0.075(0.075)(0.007)	−0.948(0.021)(0.013)	−0.235(0.086)(0.014)
5	−0.972(0.021)(0.017)	−0.008(0.089)(0.009)	−0.173(0.043)(0.010)	1.053(0.062)(0.016)	−0.359(0.046)(0.011)	0.943(0.084)(0.013)
6	−0.387(0.069)(0.025)	−0.642(0.152)(0.033)	0.554(0.073)(0.032)	0.605(0.184)(0.042)	0.333(0.051)(0.019)	0.701(0.137)(0.028)
7	0.462(0.056)(0.019)	−0.550(0.159)(0.030)	0.975(0.017)(0.008)	0.198(0.071)(0.014)	0.878(0.026)(0.015)	0.188(0.098)(0.016)
8	0.640(0.036)(0.015)	−0.399(0.099)(0.026)	0.798(0.035)(0.017)	−0.253(0.141)(0.019)	0.740(0.037)(0.014)	−0.025(0.149)(0.019)

the uncertainty on γ arising from knowledge of the charm strong phases is approximately a factor of three smaller than was possible with the CLEO measurements. For the first time, the dominant systematic uncertainty for γ measurement from the strong-phase parameters will be constrained to around 1°, or less, for γ measurements with future B experiments [15,18–21]. The predicted statistical uncertainties on γ from LHCb prior to the start of high-luminosity LHC operation in the mid 2020s, and from Belle II are expected to be around 1.5° [35,36]. The improved precision achieved here will ensure that measurements of γ from LHCb and Belle II over the next decade are not limited by the knowledge of these strong-phase parameters.

These strong-phase parameters also provide critical inputs in model-independent measurements of charm mixing and CP violation in $D^0 \to K_S^0\pi^+\pi^-$ decays [26,27]. As detailed in Ref. [26], the precision of the charm-mixing parameters x and y is dependent on c_i and s_i inputs. With 5×10^8 $D^0 \to K_S^0\pi^+\pi^-$ signal decays, which is the anticipated yield at LHCb in 2030, the uncertainty from the CLEO determination of the strong phases is expected to be approximately a factor 3.8 (5.0) larger than the statistical uncertainty for x (y) [26], leading to measurements where the overall precision is limited by the strong-phase inputs. To evaluate the impact of our c_i and s_i results on the measurements of x and y, we generate 5×10^8 $D^0 \to K_S^0\pi^+\pi^-$ signal decays using input charm-mixing parameters $x = 0.4\%$ and $y = 0.6\%$, with no CP violation. By using the "bin-flip method" [26] and keeping the c_i and s_i constrained according to our measurements, the expected statistical uncertainties on x and y are 0.027% and 0.061%, respectively. Thus, compared with the expected statistical uncertainties on x (0.034%) and y (0.091%) with CLEO inputs [26], it is clear that our results will significantly reduce uncertainties on future charm-mixing measurements.

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[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[2] J. Brod and J. Zupan, J. High Energy Phys. 01 (2014) 051.
[3] M. Blanke and A. Buras, Eur. Phys. J. C **79**, 159 (2019).
[4] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).
[5] J. Charles, A. Höcker, H. Lacker, S. Laplace, F. R. Le Diberder, J. Malclés, J. Ocariz, M. Pivk, and L. Roos (CKMfitter Group), Eur. Phys. J. C **41**, 1 (2005); and updates at http://ckmfitter.in2p3.fr; M. Bona *et al.* (UTfit Collaboration), J. High Energy Phys. 07 (2005) 028; and updates at http://utfit.org/UTfit/WebHome.
[6] J. Brod, A. Lenz, G. Tetlalmatzi-Xolocotzi, and M. Wiebusch, Phys. Rev. D **92**, 033002 (2015).
[7] M. Gronau and D. London, Phys. Lett. B **253**, 483 (1991).
[8] M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991).
[9] D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997).
[10] D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. D **63**, 036005 (2001).
[11] A. Giri, Y. Grossman, A. Soffer, and J. Zupan, Phys. Rev. D **68**, 054018 (2003).
[12] A. Bondar and A. Poluektov, Eur. Phys. J. C **47**, 347 (2006); **55**, 51 (2008).
[13] R. A. Briere *et al.* (CLEO Collaboration), Phys. Rev. D **80**, 032002 (2009).
[14] J. Libby *et al.* (CLEO Collaboration), Phys. Rev. D **82**, 112006 (2010).
[15] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 08 (2018) 176; 10 (2018) 107(E).
[16] M. Ablikim *et al.* (BESIII Collaboration), companion paper, Phys. Rev. D **101**, 112002 (2020).
[17] M. Ablikim *et al.* (BESIII Collaboration), Chin. Phys. C **37**, 123001 (2013); Phys. Lett. B **753**, 629 (2016).
[18] R. Aaij *et al.* (LHCb Collaboration), Phys. Lett. B **718**, 43 (2012).
[19] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 10 (2014) 097.
[20] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 06 (2016) 131.
[21] H. Aihara *et al.* (Belle Collaboration), Phys. Rev. D **85**, 112014 (2012).
[22] V. Vorobyev *et al.* (Belle Collaboration), Phys. Rev. D **94**, 052004 (2016).
[23] A. Bondar, A. Kuzmina, and V. Vorobyev, J. High Energy Phys. 03 (2018) 195.
[24] C. Thomas and G. Wilkinson, J. High Energy Phys. 10 (2012) 185.
[25] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 04 (2016) 033.
[26] A. Di Canto, J. Garra Ticó, T. Gershon, N. Jurik, M. Martinelli, T. Pilař, S. Stahl, and D. Tonelli, Phys. Rev. D **99**, 012007 (2019).
[27] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **122**, 231802 (2019).
[28] M. Ablikim *et al.* (BESIII Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A **614**, 345 (2010).
[29] S. Malde, C. Thomas, G. Wilkinson, P. Naik, C. Prouve, J. Rademacker, J. Libby, M. Nayak, T. Gershon, and R. A. Briere, Phys. Lett. B **747**, 9 (2015).
[30] I. Adachi *et al.* (*BABAR* and Belle Collaborations), Phys. Rev. D **98**, 112012 (2018).
[31] S. Harnew, P. Naik, C. Prouve, J. Rademacker, and D. Asner, J. High Energy Phys. 01 (2018) 144.
[32] T. Evans, S. T. Harnew, J. Libby, S. Malde, J. Rademacker, and G. Wilkinson, Phys. Lett. B **757**, 520 (2016).
[33] J. Insler *et al.* (CLEO Collaboration), Phys. Rev. D **85**, 092016 (2012).
[34] P. K. Resmi, J. Libby, S. Malde, and G. Wilkinson, J. High Energy Phys. 01 (2018) 082.
[35] I. Bediaga *et al.* (LHCb Collaboration), Report Nos. HCB-PUB-2018-009, CERN-LHCC-2018-027, arXiv:1808.08865.
[36] E. Kou *et al.* (Belle II Collaboration), Prog. Theor. Exp. Phys. **123C01**, 029201 (2020).